

1. Differentiate each of the following:

- (a) $\tan^{-1}(e^x)$ that is $\text{Arctan}(e^x)$
- (b) $\sin^{-1}(1 + x^2)$ that is $\text{Arcsin}(1 + x^2)$
- (c) e^{x^3+x}

2. Integrate each of the following:

- (a) $\int \sin^3 x \cos^2 x \, dx$
- (b) $\int \sec x \, dx$
- (c) $\int \sin^2 x \, dx$
- (d) $\int x \sin(x) \, dx$
- (e) $\int \frac{dx}{(x^2 + 9)}$
- (f) $\int \frac{1}{\sqrt{9 - x^2}} \, dx$
- (g) $\int \frac{\ln(x)}{x} \, dx$
- (h) $\int \ln x \, dx$
- (i) $\int xe^x \, dx$
- (j) $\int xe^{x^2} \, dx$

$$\text{Hints: } \cos^2 x = \frac{1 + \cos 2x}{2},$$

$$\sin^2 x = \frac{1 - \cos 2x}{2},$$

$$1 + \tan^2 x = \sec^2 x,$$

$$\sin 2x = 2 \sin x \cos x$$

- (a) Consider the parametric curve

$$x = a \cos t, \quad y = a \sin t, \quad t = 0 \dots 2\pi$$

- i. Sketch this curve.
- ii. Consider the parametric curve

$$x = \sin^2 t, \quad y = 2 \cos^2 t, \quad t = 0 \dots 2\pi$$

- iii. Sketch this curve.

- (b) **For each of the following:**

- i. Shade the area in question;
- ii. find the points of intersection (if needed);
- iii. set up the integral for the area;
- iv. evaluate the area;

- (c) The region bounded by $r = 2 + 2 \sin(\theta)$
- (d) The region bounded *INSIDE* $r = 2 \sin(\theta)$ and *OUTSIDE* $r = 1$.
- (e) The region bounded by *BOTH* $r = \sin(\theta)$ and $r = \cos(\theta)$
- (f) The region bounded by $r = \cos(2\theta)$.

Math 1572

Calculus II : Exam 3

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- (a) . **Answer the following questions, giving some valid reasons to support your answer, not just answers.**

- i. For $a_n = (-1)^n \frac{\ln n}{n}$, find $\lim_{n \rightarrow \infty} a_n$?
- ii. For $a_n = (n - 1)/n$, find $\lim_{n \rightarrow \infty} a_n$?
- iii. Find $\lim_{n \rightarrow \infty} \frac{1 + 2^n}{3^n}$
- iv. Find $\lim_{n \rightarrow \infty} \frac{2^{2n}}{3^n}$?

- (b) **For each of the following series determine if the series converges conditionally, converges absolutely or diverges.**

$$\text{i. } \sum_{n=1}^{+\infty} (-1)^n \frac{\ln(n)}{n}$$

$$\text{ii. } \sum_{n=1}^{+\infty} \frac{2^n}{n!}$$

$$\text{iii. } \sum_{n=0}^{\infty} \frac{2}{4n^2 - 1}$$

$$\text{iv. } \sum_{n=1}^{+\infty} \frac{(-1)^n n}{3^n}$$

$$\text{v. } \sum_{n=1}^{+\infty} \frac{(-1)^n (2n + 1)}{(5n + 7)}$$

- (c) **For each of the following power series determine where the series converges or diverges, and determine the radius of convergence, also decide if the series converges at the ends and what type.**

$$\text{i. } \sum_{n=1}^{+\infty} \frac{2^n x^n}{n^2}$$

$$\text{ii. } \sum_{n=1}^{+\infty} \frac{(-1)^n (x^n)}{n^2}$$

3. **NEW STUFF** Represent the following series then diff and int them and describe their region of convergence.

$$\text{(a) } y = \frac{1}{1+x^2}$$

$$\text{(b) } y = \frac{1}{x+1}$$

$$\text{(c) } y = \ln(x-2)$$