restart:
with (student): with(plots):

Warning, the name changecoords has been redefined

Ex11

p1 := plot([sqrt(4*cos(2*theta)), theta, theta=0..2*Pi], coords=polar, color = red): p2 := plot(x, x = -Pi..Pi, color=blue): display(p1, p2);
note there are 2 halves which are themselves symmetric so the area in question is 4 x the area of that one part.

> 4*Int(1/2*(4*cos(2*theta)),theta=0 .. Pi/4)=4*1/2*int((4*cos(2*theta)),theta=0 .. Pi/4);

\[
4 \int_0^{\pi/4} 2 \cos(2 \theta) \, d\theta = 4
\]

Ex21 find area of inner loop
note there is 1 inner loop bubble so the area in question is 2 x the area of that one part half that part which obviously has an intersection at Pi/4

\[ 2 \int_{\pi/4}^{3\pi/4} \frac{1}{2} (1 + 2 \sin(\theta))^2 \, d\theta \]
\[ 3 \pi/2 = 2 \int_{\pi/4}^{3\pi/4} (1 + 2 \sin(\theta))^2 \, d\theta \]
\[ 3 \pi/2 \text{evalf}() \]
\[ \int_{\frac{\pi}{6}}^{\frac{3\pi}{2}} \frac{1}{2} (1 + 2 \sin(\theta))^2 \, d\theta = -\frac{3\sqrt{3}}{2} + \pi \]

\[
\begin{align*}
2 \int_{0}^{\frac{\pi}{2}} (1 + 2 \sin(\theta))^2 \, d\theta &= -\frac{3\sqrt{3}}{2} + \pi \\
&= 0.5435164422 = 0.543516442
\end{align*}
\]

Ex 27

\[
\begin{align*}
\text{eq} &:= 3 \cos(\theta) - (1 + \cos(\theta)) \\
\text{solve(eq, theta)}
\end{align*}
\]

\[
\begin{align*}
eq &:= 2 \cos(\theta) - 1 \\
&= \frac{\pi}{3}
\end{align*}
\]
\[ 2 \cdot \left( \int_{0}^{\pi/3} \frac{1}{2} (3 \cos(\theta))^2 \, d\theta - \int_{0}^{\pi/3} \frac{1}{2} (1 + \cos(\theta))^2 \, d\theta \right) = 2 \cdot \int_{0}^{\pi/3} \left( \frac{1}{2} (3 \cos(\theta))^2 - (1 + \cos(\theta))^2 \right) \, d\theta = \pi \]

Ex31

\[ > p1 := \text{plot}([\sin(2*\theta), \theta, \theta=0..2*\Pi], \text{coords}=\text{polar}, \text{color}=\text{red}); \]
\[ > p2 := \text{plot}([\cos(2*\theta), \theta, \theta=0..2*\Pi], \text{coords}=\text{polar}, \text{color}=\text{blue}); \]
\[ > \text{display}(p1, p2); \]

note we are interested in inside both red and blue so the area in question is sixteen times the partial leaf. our first step is to solve the intersection

\[ > \text{eq} := \cos(2*\theta) - \sin(2*\theta); \]
\[ > \text{solve(eq, theta)}; \]

\[ eq := \cos(2 \theta) - \sin(2 \theta) \]

\[ \frac{\pi}{8} \]

\[ > 16 \times \left( \text{int}\left( \frac{1}{2} \sin^2(2\theta), \theta = 0 \ldots \frac{\pi}{8} \right) \right) = 16 \times \left( \text{int}\left( \frac{1}{2} \sin^2(2\theta), \theta = 0 \ldots \frac{\pi}{8} \right) \right); \]

\[ 16 \int_{0}^{\frac{\pi}{8}} \frac{1}{2} \sin^2(2\theta) \, d\theta = -1 + \frac{\pi}{2} \]