restart:

with (student): with(plots):

Ex1

plot([cos(t), sin(t), t=0..2*Pi]);
\[
x := t \rightarrow a \cos(t); \\
y := t \rightarrow a \sin(t);
\]
> \[ \text{Int} \left( \sqrt{ (\text{Diff}(x(t),t))^2 + (\text{Diff}(y(t),t)^2) } \right), t=0 \ldots 2\pi \] = \text{int} \left( \sqrt{ \left( \frac{\partial}{\partial t} (a \cos(t)) \right)^2 + \left( \frac{\partial}{\partial t} (a \sin(t)) \right)^2 } \right) dt = 2 \pi a \text{csgn}(a) \]  

(2)

Ex2

> > \[ \text{Int} \left( \sqrt{ (\text{Diff}(x(t),t))^2 + (\text{Diff}(y(t),t)^2) } \right), t=0 \ldots \pi/2 \] = \text{int} \left( \sqrt{ \left( \frac{\partial}{\partial t} (a \cos(t)) \right)^2 + \left( \frac{\partial}{\partial t} (a \sin(t)) \right)^2 } \right) dt = 2 \pi a \text{csgn}(a) \]

Ex3

> \[ r := \text{theta} \rightarrow 2 + 2 \sin(\text{theta}); \text{plot}([r(\text{theta}), \text{theta}, \text{theta}=0 \ldots \pi]) \]
\[ \theta \rightarrow 2 + 2 \sin(\theta) \]
\[ 2 \times \frac{1}{2} \cdot \int (r(\theta))^2, \theta = -\frac{\pi}{2} \ldots \frac{\pi}{2} = 2 \times \frac{1}{2} \cdot \int (r(\theta))^2, \theta = -\frac{\pi}{2} \ldots \frac{\pi}{2} \]
\[
\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(2 + 2 \sin(\theta)\right)^2 \, d\theta = 6 \pi
\]

Ex4

\[
> 2 \times \frac{1}{2} \times \int (r(\theta))^2, \theta = -\frac{\pi}{2} \ldots \frac{\pi}{2} = 2 \times \frac{1}{2} \times \int (r(\theta))^2, \theta = -\frac{\pi}{2} \ldots \frac{\pi}{2} ;
\]

\[
\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(2 + 2 \sin(\theta)\right)^2 \, d\theta = 6 \pi
\]
\[ 2 \times \frac{1}{2} \int (r_1^2 - r_2^2, \theta = \frac{\pi}{6} \ldots \frac{\pi}{2}) = 2 \times \frac{1}{2} \int ((r_1(\theta))^2 - (r_2(\theta))^2, \theta = \frac{\pi}{6} \ldots \frac{\pi}{2}) \]
\[ \int_{\frac{1}{2}\pi}^{\frac{1}{6}\pi} (r_1^2 - r_2^2) \, d\theta = \frac{1}{2} \sqrt{3} + \frac{1}{3} \pi \]  

Ex5

\[ p_1 := \text{plot}([\sin(t), t, t=0..2*\pi], \text{coords=polar}, \text{color=red}) : \]

\[ p_2 := \text{plot}([\cos(t), t, t=0..2*\pi], \text{coords=polar}, \text{color=blue}) : \]

\[ p_1 := \text{plot}([\sin(t), t, t=0..2*\pi], \text{coords=polar}, \text{color=red}) : \]

\[ \text{display}(p1, p2); \]

\[ 2 \int_{\frac{1}{4}\pi}^{\frac{1}{8}\pi} 2 \left( \int_{0}^{\frac{1}{2}\pi} \frac{1}{2} \sin(\theta)^2 \, d\theta \right) = -\frac{1}{4} + \frac{1}{8} \pi \]  

Ex6

\[ \text{plot}([\cos(2*theta), \theta, \theta=0..2*\pi], \text{coords=polar}, \text{color=red}); \]
note there are 4 leaves
\[
8 \left( \int_0^{\pi/4} \frac{1}{2} \cos(2\theta)^2 \, d\theta \right) = \frac{1}{2} \pi \quad (7)
\]