```
 restart:with (student):
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```
# question 1
```

```
a(n) := 1 - 1/n:
Limit(a(n), n=infinity)=limit(a(n), n = infinity);
```

$$\lim_{n \to \infty} \left(1 - \frac{1}{n}\right) = 1 \qquad (1)$$

```
a(n) :=  ( - 1)^n:
Limit(a(n), n=infinity)=limit(a(n), n = infinity);
```

$$\lim_{n \to \infty} (-1)^n = -1 .. 1 \qquad (2)$$

```
# so no limit
```

```
a(n) :=  (1+2^n)/(1 - 3^n):
Limit(a(n), n=infinity)=limit(a(n), n = infinity);
```

$$\lim_{n \to \infty} \left(\frac{1 + 2^n}{1 - 3^n}\right) = 0 \qquad (3)$$

(4)

```
a(n) :=  ( 1/ 3)^n:
Limit(a(n), n=infinity)=limit(a(n), n = infinity);
```

$$\lim_{n \to \infty} \left(\frac{1}{3}\right)^n = 0 \qquad (5)$$

```
# 7a
```

(6)

```
Sum(' (-1)^n*ln(n)/(n)', 'n'=1..N);# diverges
```

$$\sum_{n=1}^{N} \frac{(-1)^n \ln(n)}{n} \qquad (7)$$

```
a(n) :=(-1)^n*ln(n)/(n);
```

$$a(n) := \frac{(-1)^n \ln(n)}{n} \qquad (8)$$

try integral testt

```
g:= x -> ln( x)/x:
Limit(Int(g(x),x=1 ..X),X=infinity)= limit(int(g(x),x=1 ..X),X=
infinity);
```

$$\lim_{X \to \infty} \left(\int_1^X \frac{\ln(x)}{x}\, dx \right) = \infty \quad (9)$$

so the

```
Limit(a(n), n=infinity)=limit(a(n), n = infinity); # nth term goes to zero
```

$$\lim_{n \to \infty} \left(\frac{(-1)^n \ln(n)}{n} \right) = 0 \quad (10)$$

```
diff(g(x),x); # which is negative for x >= e
```

$$\frac{1}{x^2} - \frac{\ln(x)}{x^2} \quad (11)$$

```
Sum(' ln(n)/(n)', 'n'=1..N);# yields conditional convergence
```

$$\sum_{n=1}^{N} \frac{\ln(n)}{n} \quad (12)$$

7b

```
a(n) := 1/(n + n ^2);
```

$$a(n) := \frac{1}{n + n^2} \quad (13)$$

```
Sum( a(n), 'n'=1..N); B(N) := sum('a(n)', 'n'=1..N); Limit(B(N), N=infinity)= limit(B(N), N=infinity);
```

$$\sum_{n=1}^{N} \frac{1}{n + n^2}$$

$$B(N) := \frac{N(2 + N)}{2 + 3N + N^2}$$

$$\lim_{N \to \infty} \left(\frac{N(2 + N)}{2 + 3N + N^2} \right) = 1 \quad (14)$$

limit compare it with 1/ n^2

```
b(n) := 1/n^2;
```

$$b(n) := \frac{1}{n^2} \quad (15)$$

```
Limit(a(n)/b(n),n=infinity)= limit(a(n)/b(n),n=infinity);
```

$$\lim_{n \to \infty} \left(\frac{n^2}{n + n^2} \right) = 1 \quad (16)$$

 # so it converges by limit comparison

```
# 7c
```

```
a(n) := n/sqrt(4 + n ^5); Sum( a(n), 'n'=1..N); B(N) := sum(a(n)
, 'n'=1..N); Limit(B(N),N=infinity)= limit(B(N), N=infinity);
```

$$\sum_{n=1}^{N} \frac{n}{\sqrt{4+n^5}}$$

$$B(N) := \sum_{n=1}^{N} \frac{n}{\sqrt{4+n^5}}$$

$$\lim_{N\to\infty} \left(\sum_{n=1}^{N} \frac{n}{\sqrt{4+n^5}} \right) = \sum_{n=1}^{\infty} \frac{n}{\sqrt{4+n^5}} \tag{17}$$

limit compare it with 1/ n^3/2

```
 b(n) := 1/n^(3/2);# a known convergent series
```

$$b(n) := \frac{1}{n^{3/2}} \tag{18}$$

```
Limit(a(n)/b(n),n=infinity)= limit(a(n)/b(n),n=infinity);
```

$$\lim_{n\to\infty} \left(\frac{n^{5/2}}{\sqrt{4+n^5}} \right) = 1 \tag{19}$$

```
# so it converges
```

7d

```
a(n) := (abs(1 - 3*n)/(3 + 4*n))^n;
```

$$a(n) := \left(\frac{|-1+3n|}{3+4n} \right)^n \tag{20}$$

```
Sum( a(n), 'n'=1..N); B(N) := sum(a(n), 'n'=1..N); Limit(B(N),N=
infinity)= limit(B(N), N=infinity);
```

$$\sum_{n=1}^{N} \left(\frac{|-1+3n|}{3+4n} \right)^n$$

$$B(N) := \sum_{n=1}^{N} \left(\frac{|-1+3n|}{3+4n} \right)^n$$

$$\lim_{N\to\infty} \left(\sum_{n=1}^{N} \left(\frac{|-1+3n|}{3+4n} \right)^n \right) = \sum_{n=1}^{\infty} \left(\frac{|-1+3n|}{3+4n} \right)^n \tag{21}$$

limit compare it with (3/4)^ n2

```
b(n) := (3/4)^n;
```

$$b(n) := \left(\frac{3}{4}\right)^n \qquad (22)$$

```
Limit(a(n)/b(n),n=infinity)= limit(a(n)/b(n),n=infinity);
```

$$\lim_{n\to\infty}\left(\frac{\left(\frac{|-1+3n|}{3+4n}\right)^n}{\left(\frac{3}{4}\right)^n}\right) = e^{-\frac{13}{12}} \qquad (23)$$

7e

```
a(n) := (2^n)/(n!);
```

$$a(n) := \frac{2^n}{n!} \qquad (24)$$

```
Sum( a(n), 'n'=1..N); B(N) := sum(a(n), 'n'=1..N); Limit(B(N),N=
infinity)= limit(B(N), N=infinity);
```

$$\sum_{n=1}^{N}\frac{2^n}{n!}$$

$$B(N) := \frac{e^2\,\Gamma(N+1, 2) - \Gamma(N+1)}{\Gamma(N+1)}$$

$$\lim_{N\to\infty}\left(\frac{e^2\,\Gamma(N+1, 2) - \Gamma(N+1)}{\Gamma(N+1)}\right) = \lim_{N\to\infty}\left(\frac{e^2\,\Gamma(N+1, 2) - \Gamma(N+1)}{\Gamma(N+1)}\right) \qquad (25)$$

use ratio test

```
b(n) := (2^(n +1))/((n+1)!);
```

$$b(n) := \frac{2^{n+1}}{(n+1)!} \qquad (26)$$

```
Limit(b(n)/a(n),n=infinity)= limit(b(n)/a(n),n=infinity);
```

$$\lim_{n\to\infty}\left(\frac{2^{n+1}\,n!}{(n+1)!\,2^n}\right) = 0 \qquad (27)$$

```
# so it converges absolutely
```

7f

```
a(n) := 2/(4*n^2-1);
```

$$a(n) := \frac{2}{4n^2 - 1} \qquad (28)$$

```
Sum( a(n), 'n'=1..N); B(N) := sum(a(n), 'n'=1..N); Limit(B(N),N=
infinity)= limit(B(N), N=infinity);
```

$$\sum_{n=1}^{N} \frac{2}{4 n^2 - 1}$$

$$B(N) := -\frac{1}{2N+1} + 1$$

$$\lim_{N \to \infty} \left(-\frac{1}{2N+1} + 1\right) = 1 \qquad (29)$$

use ratio test

```
b(n) := 2/(4*(n+1)^2-1);
```

$$b(n) := \frac{2}{4 (n+1)^2 - 1} \qquad (30)$$

```
Limit(b(n)/a(n),n=infinity)= limit(b(n)/a(n),n=infinity);
```

$$\lim_{n \to \infty} \left(\frac{4 n^2 - 1}{4 (n+1)^2 - 1}\right) = 1 \qquad (31)$$

```
# that test fails, so use a limit comparison
```

limit compare it with (1/n)^ 2

```
b(n) := (1/n)^2;
```

$$b(n) := \frac{1}{n^2} \qquad (32)$$

```
Limit(a(n)/b(n),n=infinity)= limit(a(n)/b(n),n=infinity);
```

$$\lim_{n \to \infty} \left(\frac{2 n^2}{4 n^2 - 1}\right) = \frac{1}{2} \qquad (33)$$

```
# it converges
```

this says it does not converge absolutely but we may still have cond. conv , so #

 # 7 g

```
a(n) := 2^(2*n)/(5^n);
```

$$a(n) := \frac{2^{2n}}{5^n} \qquad (34)$$

```
Sum( a(n), 'n'=1..N); B(N) := sum(a(n), 'n'=1..N); Limit(B(N),N=
infinity)= limit(B(N), N=infinity);
```

$$\sum_{n=1}^{N} \frac{2^{2n}}{5^n}$$

$$B(N) := -5 \left(\frac{4}{5}\right)^{N+1} + 4$$

(35)

$$\lim_{N \to \infty} \left(-5\left(\frac{4}{5}\right)^{N+1} + 4\right) = 4 \quad (35)$$

 # a geometric series

```
#7h
g:= x ->1/(x* ln( x));Int(g(x),x)= int(g(x),x);
Limit(Int(g(x),x=2 ..X),X=infinity)= limit(int(g(x),x=2 ..X),X=
infinity);
```

$$g := x \to \frac{1}{x \ln(x)}$$

$$\int \frac{1}{x \ln(x)}\, dx = \ln(\ln(x))$$

$$\lim_{X \to \infty} \left(\int_2^X \frac{1}{x \ln(x)}\, dx\right) = \infty \quad (36)$$

so it diverges by the integrel test ;hence we have conditional convergence

#7 i

```
a(n) := (-1)^n *ln(n)/n^3;
```

$$a(n) := \frac{(-1)^n \ln(n)}{n^3} \quad (37)$$

```
Sum( a(n) , 'n'=1..N);
```

$$\sum_{n=1}^{N} \frac{(-1)^n \ln(n)}{n^3} \quad (38)$$

```
g:= x -> ln( x)/x^3;Int(g(x),x)= int(g(x),x);
Limit(Int(g(x),x=2 ..X),X=infinity)= limit(int(g(x),x=2 ..X),X=
infinity);
```

$$g := x \to \frac{\ln(x)}{x^3}$$

$$\int \frac{\ln(x)}{x^3}\, dx = -\frac{1}{2}\, \frac{\ln(x)}{x^2} - \frac{1}{4 x^2}$$

$$\lim_{X \to \infty} \left(\int_2^X \frac{\ln(x)}{x^3}\, dx\right) = \frac{1}{8} \ln(2) + \frac{1}{16} \quad (39)$$

```
# so it absolute converges by the integral test
```

#7j

```
a(n) := (-1)^n*n/3^n;
```

$$a(n) := \frac{(-1)^n n}{3^n} \tag{40}$$

```
Sum( a(n) , 'n'=1..N);
```

$$\sum_{n=1}^{N} \frac{(-1)^n n}{3^n} \tag{41}$$

use ratio test

```
 b(n) := (n+1)/3^(n+1);
```

$$b(n) := \frac{n+1}{3^{n+1}} \tag{42}$$

```
Limit(b(n)/abs(a(n)),n=infinity)= limit(b(n)/abs(a(n)),n=
infinity);
```

$$\lim_{n \to \infty} \left(\frac{n+1}{3^{n+1} \left| \frac{(-1)^n n}{3^n} \right|} \right) = \frac{1}{3} \tag{43}$$

```
# hence absolute convergence
```

7 k

```
Sum(' 1/ln(n))', 'n'=1..N);
```

$$\sum_{n=1}^{N} \frac{\ln(n)}{n} \tag{44}$$

try integral testt

```
g:= x -> 1/ln( x):
Limit(Int(g(x),x=2 ..X),X=infinity)= limit(int(g(x),x=2 ..X),X=
infinity);
```

$$\lim_{X \to \infty} \left(\int_{2}^{X} \frac{1}{\ln(x)} \, dx \right) = \infty \tag{45}$$

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