```
> restart:with(plots):

Warning, the name changecoords has been redefined
```

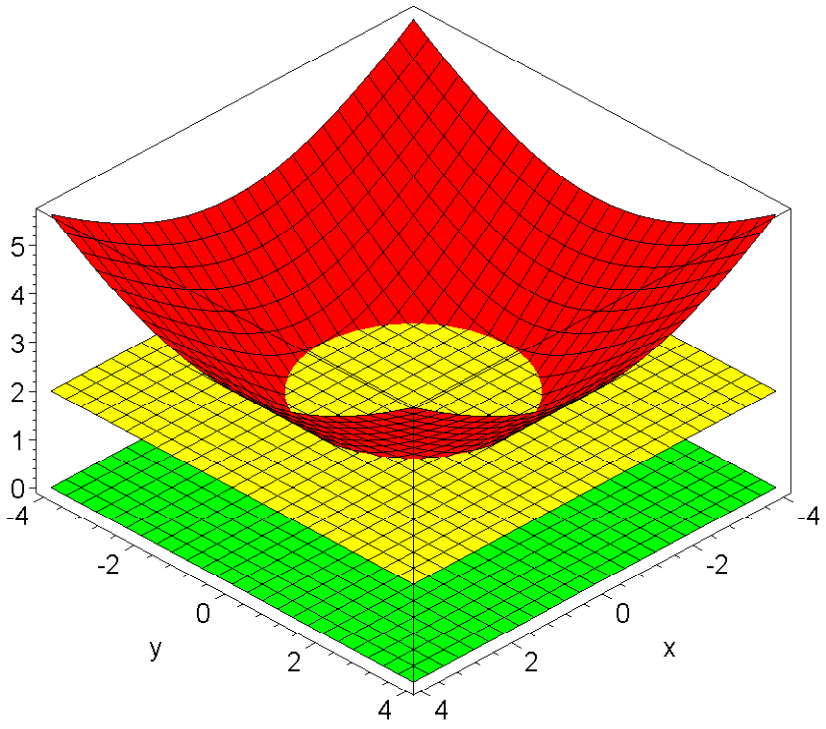
Section I ex 1

```
> g := (x, y) -> sqrt(x^2+y^2);
  h:= (x,y) -> 2; p:= (x,y) -> 0;  plot1:=plot3d(h(x,y), x = -4..4,y
  = -4 .. 4, color=yellow):plot2:= plot3d(p(x,y), x = -4..4, y = -4
  .. 4, color=green):
  plot3:=plot3d(g(x,y), x = -4..4, y = -4 .. 4, color=red):
  display3d({plot1,plot2,plot3}, axes=boxed);
```

$$g := (x, y) \to \sqrt{x^2 + y^2}$$

$$h := 2$$

$$p := 0$$



Find volume between g(x,y) and h(x,y)

```
> 4* Int(Int( 2- sqrt(x^2+y^2), y=0..sqrt(2-x^2)),x=0..sqrt(2))=

  4* int(int( 2- sqrt(x^2+y^2), y=0..sqrt(2-x^2)),x=0..sqrt(2));
  4* Int(Int(Int( 1,z =sqrt(x^2+y^2).. 2),
  y=0..sqrt(2-x^2)),x=0..sqrt(2))=

  4* int(int( int(1, z = sqrt(x^2+y^2) .. 2),
```

```
   y=0..sqrt(2-x^2)),x=0..sqrt(2));
```

$$4\int_0^{\sqrt{2}}\int_0^{\sqrt{2-x^2}} 2-\sqrt{x^2+y^2}\,dy\,dx =$$

$$4\int_0^{\sqrt{2}} -\frac{1}{2}x^2\ln(\sqrt{2-x^2}+\sqrt{2})+2\sqrt{2-x^2}-\frac{1}{2}\sqrt{2-x^2}\sqrt{2}+\frac{1}{4}x^2\ln(x^2)\,dx$$

$$4\int_0^{\sqrt{2}}\int_0^{\sqrt{2-x^2}}\int_{\sqrt{x^2+y^2}}^{2} 1\,dz\,dy\,dx =$$

$$4\int_0^{\sqrt{2}} -\frac{1}{2}x^2\ln(\sqrt{2-x^2}+\sqrt{2})+2\sqrt{2-x^2}-\frac{1}{2}\sqrt{2-x^2}\sqrt{2}+\frac{1}{4}x^2\ln(x^2)\,dx$$

TROUBLE! so try polars/cylindricals

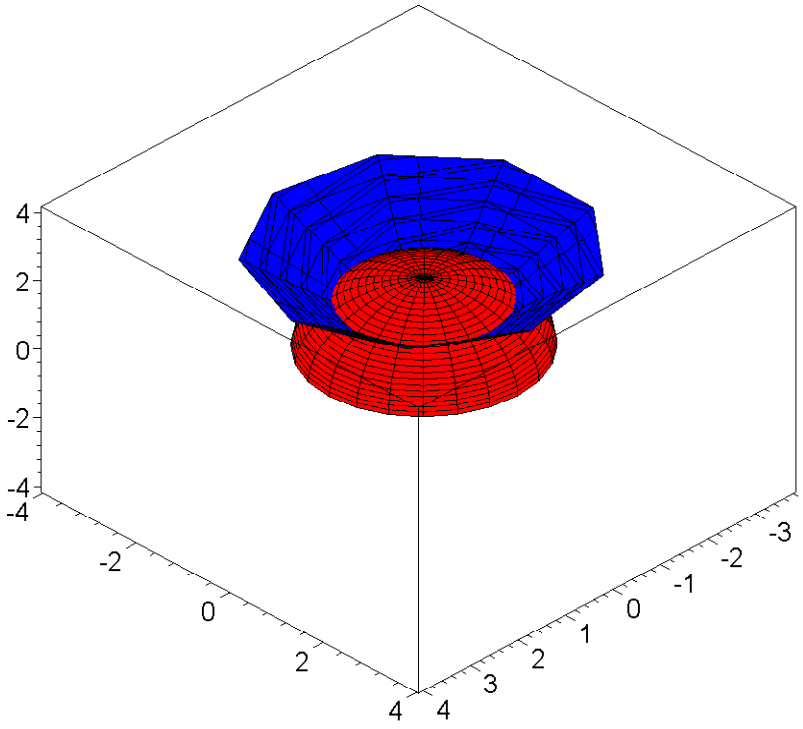
```
> Int(Int((2 - r)*r, r= 0..2),theta =0..2*Pi)=
  int(int((2 - r)*r, r= 0..2),theta =0..2*Pi);
  Int(Int( Int ( r,z = r .. 2), r= 0..2),theta =0..2*Pi)=
  int(int( int ( r,z = r .. 2), r= 0..2),theta =0..2*Pi);
>
```

$$\int_0^{2\pi}\int_0^{2}(2-r)\,r\,dr\,d\theta=\frac{8}{3}\pi$$

$$\int_0^{2\pi}\int_0^{2}\int_r^{2} r\,dz\,dr\,d\theta=\frac{8}{3}\pi$$

ex 2 vol bounded by cone angel pi/4 and sphere of radius 2 great place for sphereical coordinates

```
> plot1:=plot3d(2, theta=0..2*Pi, phi=0 .. Pi/2,
  coords=spherical,color = red, axes=boxed):
> plot2:=implicitplot3d(phi=Pi/4,rho=0..4,theta=0..2*Pi,phi=0..Pi,co
  ords=spherical , color = blue,axes =
  boxed):display3d(plot1,plot2);
  Int(Int(Int(rho^2*sin(phi), rho=0..2),
  phi=0..Pi/4),theta=0..Pi*2)= int(int(int(rho^2*sin(phi),
  rho=0..2), phi=0..Pi/4),theta=0..Pi*2);
```



$$\int_0^{2\pi}\int_0^{1/4\pi}\int_0^{2} \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta = -\frac{8}{3}\sqrt{2}\,\pi + \frac{16}{3}\pi$$

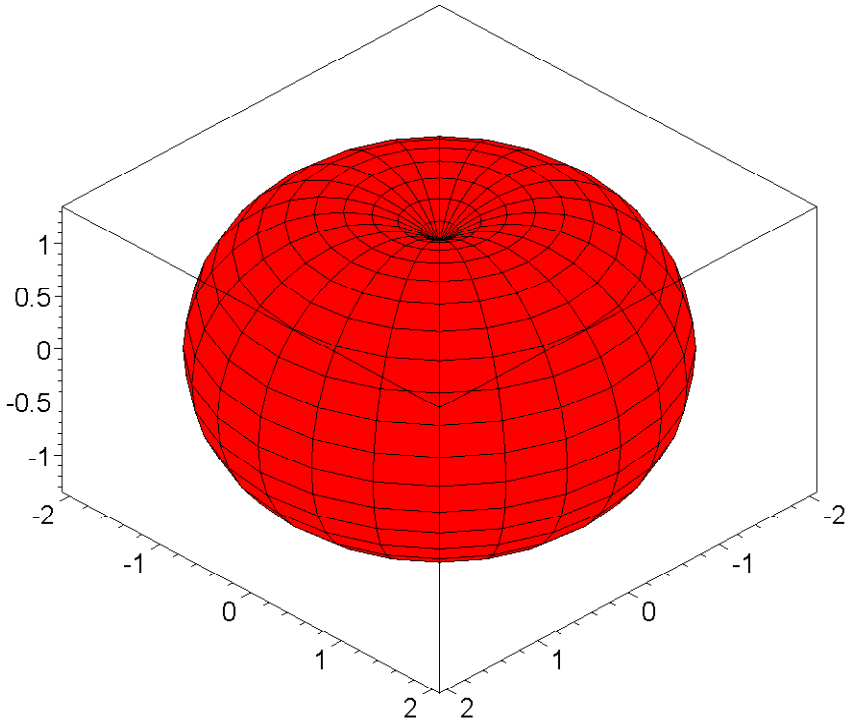
SEction II ex1

```
> Int(Int(Int(rho^2*sin(phi), rho=1..1 + sin(phi)),
  phi=0..Pi/2),theta=0..Pi*2)= int(int(int(rho^2*sin(phi), rho=1..1
  + sin(phi)), phi=0..Pi/2),theta=0..Pi*2);
```

$$\int_0^{2\pi}\int_0^{1/2\pi}\int_1^{1+\sin(\phi)} \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta = \frac{5}{8}\pi^2 + \frac{4}{3}\pi$$

but what is it

```
> plot3d(1+sin(phi), theta=0..2*Pi, phi=0..Pi,
  coords=spherical,color = red, axes=boxed);
```



ex 3

```
> Int(Int(Int(1, z=0..sqrt(1-x^2-y^2)),
  x=y..sqrt(1-y^2)),y=0..sqrt(2)/2)=int(int(int(1,
  z=0..sqrt(1-x^2-y^2)), x=y..sqrt(1-y^2)),y=0..sqrt(2)/2);
```

$$\int_0^{1/2\sqrt{2}} \int_y^{\sqrt{1-y^2}} \int_0^{\sqrt{1-x^2-y^2}} 1 \, dz \, dx \, dy = \frac{1}{12}\pi$$

so maple takes a while , let's do it in sphereical coorinates

```
> Int(Int(Int(rho^2*sin(phi), rho=0..1 ),
  phi=0..Pi/2),theta=0..Pi/4)= int(int(int(rho^2*sin(phi), rho=0..1
  ), phi=0..Pi/2),theta=0..Pi/4);
```

$$\int_0^{1/4\pi} \int_0^{1/2\pi} \int_0^{1} \rho^2 \sin(\phi) \, d\rho \, d\phi \, d\theta = \frac{1}{12}\pi$$

```
> Int(Int(Int(rho^2*sin(phi),rho = 0 .. 1),phi = 0 .. 1/2*Pi),theta
  = 0 .. 1/4*Pi) = 1/12*Pi;
```

$$\int_0^{1/4\,\pi}\int_0^{1/2\,\pi}\int_0^{1}\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta=\frac{1}{12}\pi$$

so what is it in sphericals

```
> plot3d(1, theta=0..Pi/4, phi=0..Pi/2, coords=spherical,color =
  red, axes=boxed);
```

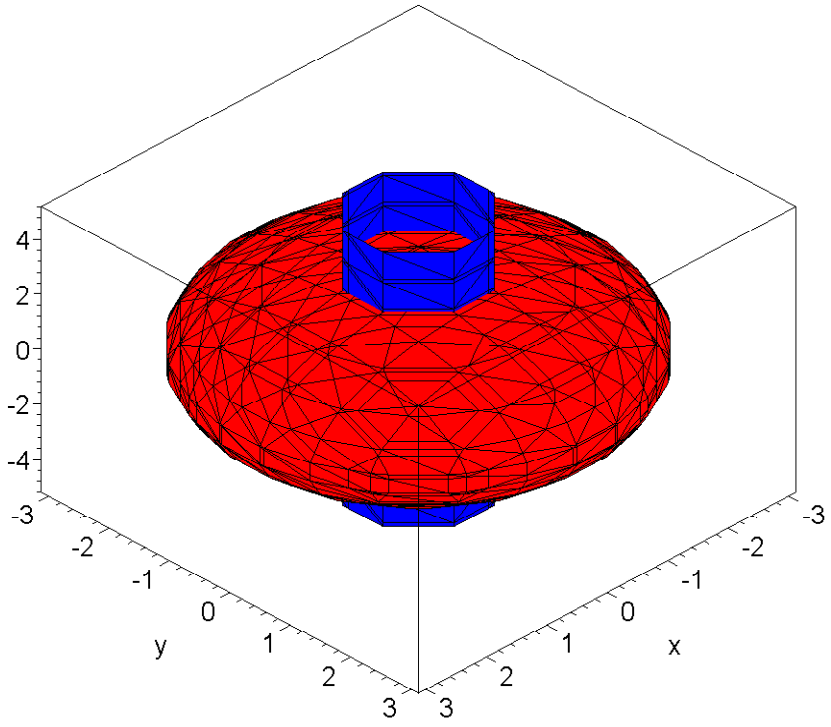
ex

```
> Int(Int(Int(rho^2*sin(phi), rho=0 ..2),  phi=0..Pi/2),theta
  =0..Pi)=
  int(int(int(rho^2*sin(phi), rho=0 ..2),  phi=0..Pi/2),theta
  =0..Pi);
```

$$\int_0^{\pi}\int_0^{1/2\,\pi}\int_0^{2}\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta=\frac{8}{3}\pi$$

ex6

```
> plot1:=implicitplot3d( x^2 + y^2 + z^2=9,x=-3..3,y=-3..3,
  z=-3..3,color=red): plot2 :=
  implicitplot3d( x^2 + y^2 =1,x=-3..3,y=-3..3,
  z=-5..5,color=blue):display({plot1,plot2},axes=boxed);
```



let's try cylindrical coordinates top is sphere and use sym 8 x

>

```
> 8*Int(Int(Int(r, z=0 .. sqrt(9-r^2)), r=0 .. 1),
  theta=0..Pi/2)=8*int(int(int(r, z=0 .. sqrt(9-r^2)), r=0 .. 1),
  theta=0..Pi/2);
```

$$8\int_0^{1/2\,\pi}\int_0^{1}\int_0^{\sqrt{9-r^2}} r\,dz\,dr\,d\theta = -\frac{64}{3}\sqrt{2}\,\pi + 36\,\pi$$

ex 7

```
> Int(Int(Int(rho^2*sin(phi), rho=0..2),
  phi=0..Pi/6),theta=0..2*Pi)=int(int(int(rho^2*sin(phi), rho=0..2),
  phi=0..Pi/6),theta=0..2*Pi);
```

>

>

$$\int_0^{2\,\pi}\int_0^{1/6\,\pi}\int_0^{2} \rho^2\,\sin(\phi)\,d\rho\,d\phi\,d\theta = -\frac{8}{3}\sqrt{3}\,\pi + \frac{16}{3}\,\pi$$

>