Oct 14, 2005

Math. 2673 sample Exam II

I. Answer the following:

1. Evaluate as indicated:
   a. Compute the limit along any line and state any conclusion:
   $\lim_{(x,y)->(0,0)} \dfrac{x^2y^2}{x^2+2y^2}$
   b. Compute the limit along any line and state any conclusion:
   $\lim_{(x,y)->(0,0)} \dfrac{2xy}{x^2+y^2}$
2. If $u = x^y$, show that
   $\dfrac{x}{y}\dfrac{\partial u}{\partial x} + \dfrac{1}{\ln x}\dfrac{\partial u}{\partial y} = 2u$
3. Find the maximum rate of change of $f(x,y) = x^2y + \sqrt{(}y)$ at the point $(2,1)$. In which direction does this max occur?
4. Find the local maximum and minimum values and saddle points of the given function. $f(x,y) = 3xy - x^2y - xy^2$
5. If $z = \cos(xy) + y\cos(x)$, where $x = u^2 + v$ and $y = u - v^2$, use the Chain Rule to find $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$
6. For $f(x,y) = x\exp(y) + \cos(xy)$ and $P = (2,0)$.
   a.) Find the rate of change in the $(3,-4)$ direction.
   b.) Find the gradient vector at P.
   c.) IN what direction is the maximum rate of change?
   d.) What is the maximum rate of change at P?
   e.) What is the direction of zero change at P, and what is that change at P?
7. For $x^2 + y^2 + z - 9 = 0$ and $P = (1,2,4)$.
   a.) Find the equation of the tangent plane at P.
8. For the function. $f(x,y) = -x^2 + xy - 2y - 2x - y^2 + 4$
   a.) Find all critical points.
   b.) Classify the points as to being local maximum,local minimum values or saddle points.
9. If $w = xy + z$ and $x = \cos(t), y = \sin(t), z = t$.
   a.) use **(ONLY)** the Chain Rule to find $\dfrac{dw}{dt}$
10. If $z = x^2 + y^2$, where $x = u + v$ and $y = u - v$,
    a. Compute $\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y}$.
    b. use the Chain Rule to find $\dfrac{\partial z}{\partial u}$
11. For the function. $f(x,y) = -x^2 + xy - 2y - 2x - y^2 + 4$
    a.) Find all critical points.

b.) Classify the points as to being local maximum,local minimum values or saddle points.

12.) Find the all critical points for the following function and label them as local maximum and minimum values and saddle points accordingly where
$f(x, y) = 4xy - x^4 - y^4 + 1$

13.) Find the equation of the tangent plane at the point $P = (-3, 1, -3)$ to the ellipsoid $x^2/9 + y^2 + z^2/9 = 3$

a.) Second consider $k = f(x, y, z)$ and use the formula $0 = \frac{\partial f}{\partial x}(x - x_o) + \frac{\partial f}{\partial y}(y - y_o) + \frac{\partial f}{\partial z}(z - z_o)$ where all the partials are evaluated at P_o. Hopefully you get the same answer.

14.) For $u = x^4 * y + y^2 * z^3$ and $x = r * s * e^t$, $y = r * s^2 * e^{-t}$ and $z = r * s * \sin(t)$ find $\frac{\partial u}{\partial s}$. Extra credit if you find the other 2

15.) Temperature is given by $T(x, y, z) = \frac{80}{1 + x^2 + 2y^2 + 3z^2}$ where $\bigtriangledown T = \frac{\partial T}{\partial x} i + \frac{\partial T}{\partial y} j + \frac{\partial T}{\partial z} k$

a.) In what direction does T increase most rapidly?

b.) What is that increase?

16.) For $z = x^2 + 3xy - y^2$ find the tangent plane at (2,3).

17.) Find the maximum rate of change of $f(x, y) = x^2 y + \sqrt(y)$ at the point $(2, 1)$. In which direction does this max occur?

18.) Find the local maximum and minimum values and saddle points of the given function.
$f(x, y) = 3xy - x^2 y - xy^2$

19.) If $z = \cos(xy) + y\cos(x)$, where $x = u^2 + v$ and $y = u - v^2$, use the Chain Rule to find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$