Intersecting Cylinders

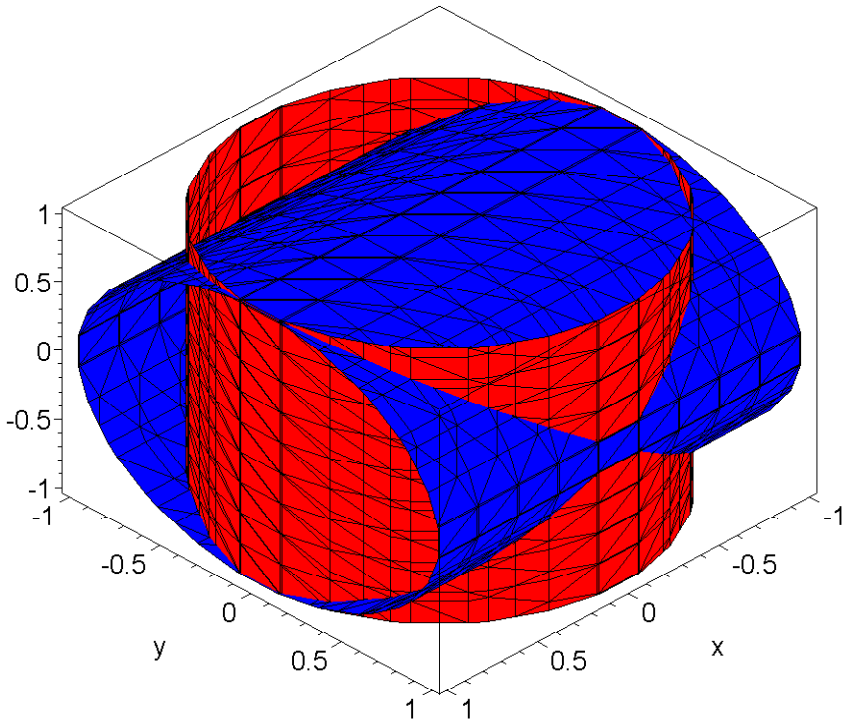
A standard problem for three dimensional integration is to find the volume of two intersecting cylinders. One hard part of this problem is seeing the shape of the intersection. We take the cylinders

$x^2 + y^2 = 1$ and $z^2 + y^2 = 1$.

In an attempt to see the intersection, we first draw the two cylinders, one red and the other blue.

```
> restart:with(plots);cyl1:=implicitplot3d(x^2 +y^2=1, x=-1.. 1,
  y=-1.. 1,z=-1 .. 1,axes=boxed,color=red):cyl2:=implicitplot3d(y^2
  +z^2=1, x=-1.. 1, y=-1.. 1,z=-1 ..
  1,axes=boxed,color=blue):display3d(cyl1,cyl2);
Warning, the name changecoords has been redefined
```

[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *cylinderplot*, *densityplot*, *display*, *display3d*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *odeplot*, *pareto*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *replot*, *rootlocus*, *semilogplot*, *setoptions*, *setoptions3d*, *spacecurve*, *sparsematrixplot*, *sphereplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*]

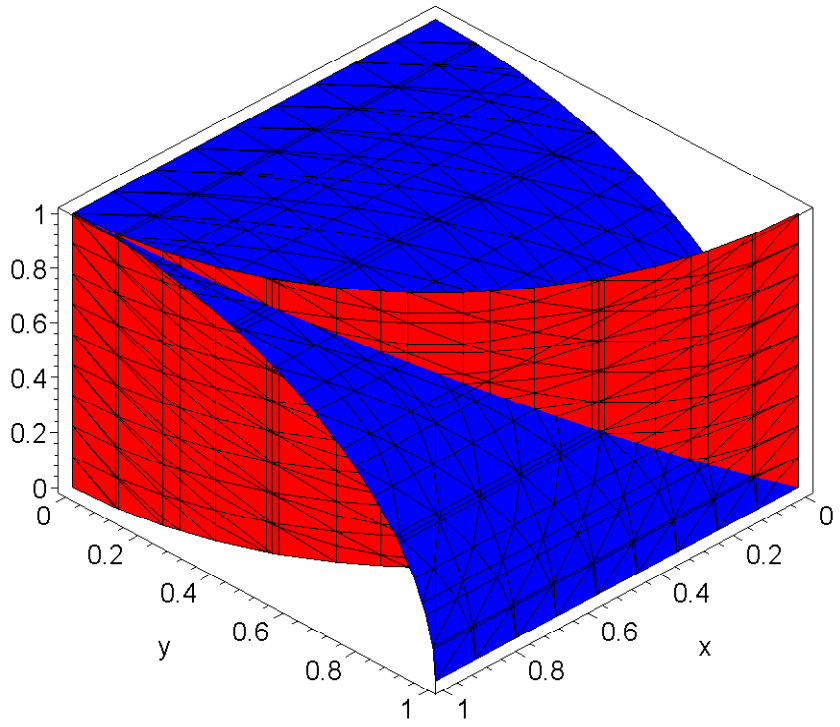


let's just look at in in the first quadrant

```
> restart:with(plots);cyl1:=implicitplot3d(x^2 +y^2=1, x=0.. 1,
  y=0.. 1,z=0 .. 1,axes=boxed,color=red):cyl2:=implicitplot3d(y^2
  +z^2=1, x=0.. 1, y=0.. 1,z=0 ..
  1,axes=boxed,color=blue):display3d(cyl1,cyl2);
```

```
Warning, the name changecoords has been redefined
```

[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *cylinderplot*, *densityplot*, *display*, *display3d*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *odeplot*, *pareto*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *replot*, *rootlocus*, *semilogplot*, *setoptions*, *setoptions3d*, *spacecurve*, *sparsematrixplot*, *sphereplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*]



Now perhaps we see the intersection which produces the base and how to compute the volume. Compute the volume indicated in the above picture and multiply by eight.and then try polars

```
> 8*Int(Int(sqrt(1-y^2),y=0..sqrt(1-x^2)),x=0..1)=
  8*Int(int(sqrt(1-y^2),y=0..sqrt(1-x^2)),x=0..1);
  8*int(int(sqrt(1-y^2),y=0..sqrt(1-x^2)),x=0..1);
```

$$8\int_0^1\int_0^{\sqrt{1-x^2}}\sqrt{1-y^2}\,dy\,dx = 8\int_0^1 \frac{1}{2}\sqrt{1-x^2}\,\sqrt{x^2}+\frac{1}{2}\arcsin(\sqrt{1-x^2})\,dx$$

$$\frac{16}{3}$$

```
> 8*Int(Int(r*sqrt(1-(r*cos(theta))^2),r=0..1),theta=0..Pi/2)=
  8*Int(int(r*sqrt(1-(r*cos(theta))^2),r=0..1),theta=0..Pi/2);
```

$$8\int_0^{1/2\,\pi}\int_0^1 r\sqrt{1-r^2\cos(\theta)^2}\,dr\,d\theta = 8\int_0^{1/2\,\pi} -\frac{1}{3}\frac{(1-\cos(\theta)^2)^{(3/2)}-1}{\cos(\theta)^2}\,d\theta$$

```
> 8*int(int(r*sqrt(1-(r*cos(theta))^2),r=0..1),theta=0..Pi/2);
```

$$\frac{16}{3}$$

```
>
>
>
>
```