```
>
> with(VectorCalculus);
```

```
Warning, the assigned names <,> and <|> now have a global binding

Warning, these protected names have been redefined and unprotected: *, +, -, ., D,
Vector, diff, int, limit, series
```

[*&x*, *, +, -, ., <,>, <|>, *AddCoordinates*, *ArcLength*, *BasisFormat*, *Binormal*, *CrossProd*, *CrossProduct*, *Curl*, *Curvature*, D, *Del*, *DirectionalDiff*, *Divergence*, *DotProd*, *DotProduct*, *Flux*, *GetCoordinateParameters*, *GetCoordinates*, *Gradient*, *Hessian*, *Jacobian*, *Laplacian*, *LineInt*, *MapToBasis*, *Nabla*, *Norm*, *Normalize*, *PathInt*, *PrincipalNormal*, *RadiusOfCurvature*, *ScalarPotential*, *SetCoordinateParameters*, *SetCoordinates*, *SurfaceInt*, *TNBFrame*, *Tangent*, *TangentLine*, *TangentPlane*, *TangentVector*, *Torsion*, *Vector*, *VectorField*, *VectorPotential*, *Wronskian*, *diff*, *evalVF*, *int*, *limit*, *series*]

```
> #
```

- The **PathInt(f, dom)** command computes the path integral of the function **f** over the path specified by **dom**. The right-hand side of **dom** is the path of integration. The left-hand side of **dom** is a list of the variables of integration.

```
> #Ex1 p 1144
> PathInt(x-3*y^2 +z, [x,y,z] = Line( <0,0,0>, <1,1,1> ),inert
  )=PathInt(x-3*y^2 +z, [x,y,z] = Line( <0,0,0>, <1,1,1> ) );
```

$$\int_0^1 (2\,t - 3\,t^2)\sqrt{3}\;dt = 0$$

```
> #Ex 2 p.1145
> PathInt(x-3*y^2 +z, [x,y,z] = LineSegments( <0,0,0>, <1,1,0>,
  <1,1,1> ),inert )=PathInt(x-3*y^2 +z, [x,y,z] = LineSegments(
  <0,0,0>, <1,1,0>, <1,1,1> ) );
```

$$\int_0^1 (t - 3\,t^2)\sqrt{2}\;dt + \int_0^1 -2 + t\;dt = -\frac{\sqrt{2}}{2} - \frac{3}{2}$$

```
>
> PathInt( y, [x,y] = Path( <cos(t),sin(t)>, t=0..Pi ),inert
  )=PathInt( y, [x,y] = Path( <cos(t),sin(t)>, t=0..Pi );
```

$$\int_0^{\pi} \sin(t)\sqrt{\sin(t)^2 + \cos(t)^2}\;dt$$

```
> #Ex 3 p. 1155
> F:=<x,y,z>;
```

$$F := x\,\mathbf{e}_{\mathbf{x}} + y\,\mathbf{e}_{\mathbf{y}} + z\,\mathbf{e}_{\mathbf{z}}$$

```
> r(t):=<cos(t),sin(t),t>;
```

$$\mathrm{r}(t) := \cos(t)\, \mathbf{e}_{\mathbf{x}} + \sin(t)\, \mathbf{e}_{\mathbf{y}} + t\, \mathbf{e}_{\mathbf{z}}$$

```
> FF:=subs( {x=cos(t), y=t,z=sin(t)}, F );
```

$$FF := \cos(t)\, \mathbf{e}_{\mathbf{x}} + t\, \mathbf{e}_{\mathbf{y}} + \sin(t)\, \mathbf{e}_{\mathbf{z}}$$

```
> diff(r(t),t);
```

$$-\sin(t)\, \mathbf{e}_{\mathbf{x}} + \cos(t)\, \mathbf{e}_{\mathbf{y}} + \mathbf{e}_{\mathbf{z}}$$

```
> DotProduct(FF,diff(r(t),t));
```

$$-\cos(t)\sin(t) + t\cos(t) + \sin(t)$$

```
> Int(%,t=0 .. Pi/2)=int(%,t=0 .. Pi/2);
```

$$\int_0^{\frac{\pi}{2}} -\cos(t)\sin(t) + t\cos(t) + \sin(t)\, dt = -\frac{1}{2} + \frac{\pi}{2}$$

```
> #Ex 2 p. 1154
> F:=<y-x^2,z-y^2,x-z^2>;
```

$$F := (y - x^2)\, \mathbf{e}_{\mathbf{x}} + (z - y^2)\, \mathbf{e}_{\mathbf{y}} + (x - z^2)\, \mathbf{e}_{\mathbf{z}}$$

```
> r(t):=<t,t^2,t^3>;
```

$$\mathrm{r}(t) := t\, \mathbf{e}_{\mathbf{x}} + t^2\, \mathbf{e}_{\mathbf{y}} + t^3\, \mathbf{e}_{\mathbf{z}}$$

```
> FF:=subs( {x=t, y=t^2,z=t^3}, F );
```

$$FF := (t^3 - t^4)\, \mathbf{e}_{\mathbf{y}} + (t - t^6)\, \mathbf{e}_{\mathbf{z}}$$

```
> diff(r(t),t);
```

$$\mathbf{e}_{\mathbf{x}} + 2\,t\, \mathbf{e}_{\mathbf{y}} + 3\,t^2\, \mathbf{e}_{\mathbf{z}}$$

```
> DotProduct(FF,diff(r(t),t));
```

$$2\,(t^3 - t^4)\,t + 3\,(t - t^6)\,t^2$$

```
> Int(%,t=0 .. Pi/2)=int(%,t=0 .. 1);
```

$$\int_0^{\frac{\pi}{2}} 2\,(t^3 - t^4)\,t + 3\,(t - t^6)\,t^2\, dt = \frac{29}{60}$$

- The **LineInt(F, dom)** command computes the line integral of the VectorField **F** over the path specified by **dom**. Note that variable names are not required here as they are in **PathInt** since they can be retrieved from coordinate system attribute of **F**.

```
> # line integral along path of {x^2+y^2=1, z=2x+4 } joining (1,0,6) to (0,1,4)
> # vector field is <z/y,0,x^2+y^2+z^2>
> SetCoordinates( 'cartesian'[x,y,z] );
```

```
> LineInt( VectorField( <z/y,0,x^2+y^2+z^2> ), Path(
  <cos(t),sin(t),2*cos(t)+4>, t=0..Pi/2 ),inert )=LineInt(
  VectorField( <z/y,0,x^2+y^2+z^2> ), Path(
  <cos(t),sin(t),2*cos(t)+4>, t=0..Pi/2 ) );
```

$$\mathit{cartesian}_{x,\,y,\,z}$$

$$\int_0^{\frac{\pi}{2}} -2\cos(t) - 4 - 2\left(\cos(t)^2 + \sin(t)^2 + (2\cos(t)+4)^2\right)\sin(t)\,dt = -\frac{164}{3} - 2\pi$$

```
> # line integral along closed curve given by 3 line segments
  joining (1,0),(0,0) to (1,1)
> # vector field is <y^2,x^2>
> SetCoordinates( 'cartesian'[x,y] );
> LineInt( VectorField( <y^2,x^2> ), LineSegments( <0,0>, <1,0>,
  <1,1> ,<0,0>),inert)=LineInt( VectorField( <y^2,x^2> ),
  LineSegments( <0,0>, <1,0>, <1,1> ,<0,0>));# closed curve
```

$$\mathit{cartesian}_{x,\,y}$$

$$\int_0^1 0\,dt + \int_0^1 1\,dt + \int_0^1 -2(1-t)^2\,dt = \frac{1}{3}$$

```
> # switch orientation
> LineInt( VectorField( <y^2,x^2> ), LineSegments( <0,0>, <1,1>,
  <1,0> ,<0,0>));# not independent of path
```

$$\frac{-1}{3}$$

```
> # line integral along  curve given by y=x^2, z=x-1 from (1,1,0) to
  (2,4,1)
> # vector field is <(x^3-2*y^2)/(x^3*y),(y^2-x^3)/(x^2*y^2),2*z^2>
> SetCoordinates( 'cartesian'[x,y,z] );
```

$$\mathit{cartesian}_{x,\,y,\,z}$$

```
> LineInt( VectorField(
  <(x^3-2*y^2)/(x^3*y),(y^2-x^3)/(x^2*y^2),2*z^2> ), Path(
  <t,t^2,t-1> ,t=1 .. 2),inert)=LineInt( VectorField(
  <(x^3-2*y^2)/(x^3*y),(y^2-x^3)/(x^2*y^2),2*z^2> ), Path(
  <t,t^2,t-1> ,t=1 .. 2));# closed curve
```

$$\int_1^2 \frac{t^3 - 2t^4}{t^5} + \frac{2(t^4 - t^3)}{t^5} + 2(t-1)^2\,dt = \frac{1}{6}$$

```
>
> (M,d):=Jacobian(
  [rho*cos(theta)*sin(phi),rho*sin(theta)*sin(phi),rho*cos(phi)],
```

```
[rho,phi,theta],'determinant' );
```

$$M, d := \begin{bmatrix} \cos(\theta)\sin(\phi) & \rho\cos(\theta)\cos(\phi) & -\rho\sin(\theta)\sin(\phi) \\ \sin(\theta)\sin(\phi) & \rho\sin(\theta)\cos(\phi) & \rho\cos(\theta)\sin(\phi) \\ \cos(\phi) & -\rho\sin(\phi) & 0 \end{bmatrix},$$

$$\cos(\theta)^2\sin(\phi)^3\rho^2 + \sin(\theta)^2\sin(\phi)^3\rho^2 + \rho^2\cos(\theta)^2\cos(\phi)^2\sin(\phi) + \rho^2\sin(\theta)^2\cos(\phi)^2\sin(\phi)$$

```
> simplify(d,trig);
```

$$\rho^2\sin(\phi)$$

```
> (M1,d1):=Jacobian(
  [sqrt(x^2+y^2+z^2),arccos(z/sqrt(x^2+y^2+z^2)),arctan(y/x)],
  [x,y,z],'determinant' );
```

$M1, d1 :=$

$$\left[\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}}\right]$$

$$\left[\frac{z\,x}{(x^2+y^2+z^2)^{(3/2)}\sqrt{1-\frac{z^2}{x^2+y^2+z^2}}}, \frac{z\,y}{(x^2+y^2+z^2)^{(3/2)}\sqrt{1-\frac{z^2}{x^2+y^2+z^2}}}, -\frac{-\frac{z^2}{(x^2+y^2+z^2)^{(3/2)}}+\frac{1}{\sqrt{x^2+y^2+z^2}}}{\sqrt{1-\frac{z^2}{x^2+y^2+z^2}}}\right]$$

$$\left[-\frac{y}{x^2\left(1+\frac{y^2}{x^2}\right)}, \frac{1}{x\left(1+\frac{y^2}{x^2}\right)}, 0\right], \frac{1}{\sqrt{\frac{x^2+y^2}{x^2+y^2+z^2}}\,(x^2+y^2+z^2)}$$

```
> simplify(d1);
```

$$\frac{1}{\sqrt{\frac{x^2+y^2}{x^2+y^2+z^2}}\,(x^2+y^2+z^2)}$$

```
>
```