Nov 15,2006

A standard problem for three dimensional integration is to find the volume of

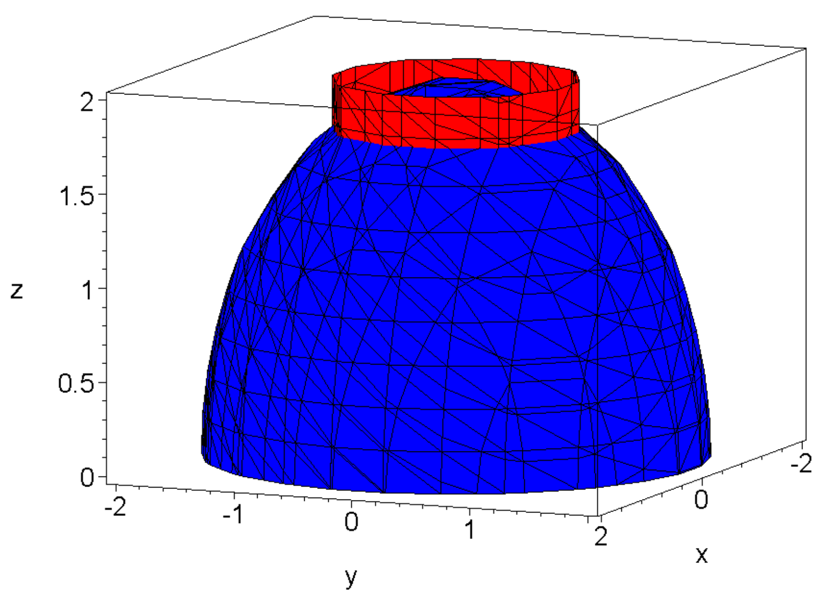
$x^2 + y^2 = 1$ and x^2 + y^2 + z^2 = 4.

In an attempt to see the intersection, we first draw the two cylinders, one red and the other blue.

```
> restart:with(plots):with(student):
Warning, the name changecoords has been redefined
```

```
> J:=implicitplot3d({ y^2 + x^2 =1},x=-2..2,y=-2..2,
  z=0..2,axes=boxed,color=red):K:=implicitplot3d(z^2+ y^2 + x^2
  =4,x=-2..2,y=-2..2,
  z=0..2,axes=boxed,color=blue):
```

```
> display3d({J,K});
```



Look at that last graph and see if the following three pictures do not give visualization to the intersection. The first picture is from the perspective of standing in the first octant with the same orientation as above..

```
>
```

```
> 8*Int(Int(Int( r, z=0..sqrt(4-r^2)
  ),r=0..1),theta=0..Pi/2)=8*int(int(int( r, z=0..sqrt(4-r^2)
```

```
),r=0..1),theta=0..Pi/2);evalf(%);#easy way in cylindricals
```

$$8\int_0^{\frac{\pi}{2}}\int_0^1\int_0^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta = -4\sqrt{3}\,\pi + \frac{32}{3}\pi$$

$$11.74472926 = 11.74472927$$

```
> 8*Int(Int(Int( 1, z=0..sqrt(4-x^2 -y^2)
),x=0..sqrt(1-y^2)),y=0..1)=8*int(int(int( 1, z=0..sqrt(4-x^2
-y^2) ),x=0..sqrt(1-y^2)),y=0..1);evalf(%);#in rect's

>

>
```

$$8\int_0^1\int_0^{\sqrt{1-y^2}}\int_0^{\sqrt{4-x^2-y^2}} 1\,dz\,dx\,dy = -4\sqrt{3}\,\pi + \frac{32}{3}\pi$$

$$11.74472926 = 11.74472927$$

```
> Int(Int(Int( rho^2*sin(phi), rho=0..2),phi=0..Pi/6),theta=0..
Pi/2)=int(int(int( rho^2*sin(phi),
rho=0..2),phi=0..Pi/6),theta=0.. Pi/2); ans1:=eval(%);

>
```

$$\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{6}}\int_0^2 \rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta = \frac{4}{3}\pi - \frac{2}{3}\sqrt{3}\,\pi$$

$$ans1 := \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{6}}\int_0^2 \rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta = \frac{4}{3}\pi - \frac{2}{3}\sqrt{3}\,\pi$$

```
> Int(Int(Int( rho^2*sin(phi),
rho=0..csc(phi)),phi=Pi/6..Pi/2),theta=0.. Pi/2)=int(int(int(
rho^2*sin(phi), rho=0..csc(phi)),phi=Pi/6..Pi/2),theta=0..
Pi/2);ans2:=eval(%);
```

$$\int_{0}^{\frac{\pi}{2}} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \int_{0}^{\csc(\phi)} \rho^2 \sin(\phi) \, d\rho \, d\phi \, d\theta = \frac{\sqrt{3}\,\pi}{6}$$

$$ans2 := \int_{0}^{\frac{\pi}{2}} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \int_{0}^{\csc(\phi)} \rho^2 \sin(\phi) \, d\rho \, d\phi \, d\theta = \frac{\sqrt{3}\,\pi}{6}$$

```
> 8*eval(ans1+ans2);
```

$$8 \int_{0}^{\frac{\pi}{2}} \int_{0}^{\frac{\pi}{6}} \int_{0}^{2} \rho^2 \sin(\phi) \, d\rho \, d\phi \, d\theta + 8 \int_{0}^{\frac{\pi}{2}} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \int_{0}^{\csc(\phi)} \rho^2 \sin(\phi) \, d\rho \, d\phi \, d\theta = \frac{32}{3}\pi - 4\sqrt{3}\,\pi$$

```
> evalf(%);
```

$$11.74472927 = 11.74472927$$

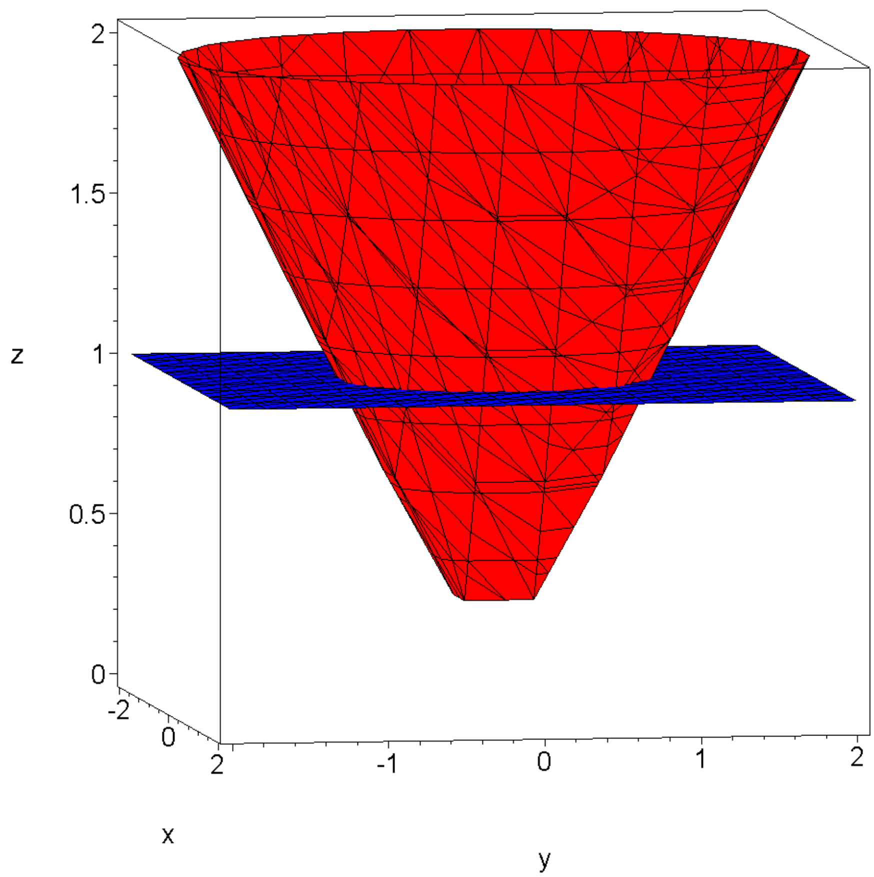
volume of a cone z=sqrt(x^1+y^2) with height 2

```
> restart:with(plots):with(student):
```

```
Warning, the name changecoords has been redefined
```

```
> J:=implicitplot3d({ sqrt(y^2 + x^2) =z},x=-2..2,y=-2..2,
  z=0..2,axes=boxed,color=red):K:=implicitplot3d(z
  =1,x=-2..2,y=-2..2,
  z=0..2,axes=boxed,color=blue):
```

```
> display3d({J,K});
```




```
> Int(Int(Int( rho^2*sin(phi),
rho=0..sec(phi)),phi=0..Pi/4),theta=0.. 2*Pi)=int(int(int(
rho^2*sin(phi), rho=0..sec(phi)),phi=0..Pi/4),theta=0..
2*Pi);evalf(%);
```

$$\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\sec(\phi)} \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta = \frac{\pi}{3}$$

$$ans1 := \int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\sec(\phi)} \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta = \frac{\pi}{3}$$

```
> Int(Int(Int( r, z=r..1) ,r=0..1),theta=0..2*Pi)=int(int(int( r,
  z=r..1 ),r=0..1),theta=0..2*Pi);evalf(%);#easy way in cylindricals
```

$$\int_0^{2\pi} \int_0^1 \int_r^1 r\, dz\, dr\, d\theta = \frac{\pi}{3}$$

$$1.047197551 = 1.047197551$$

```
> 4*Int(Int(Int( 1, z=sqrt(x^2 +y^2)..1
  ),x=0..sqrt(1-y^2)),y=0..1)=4*int(int(int( 1, z=sqrt(x^2 +y^2)..1
  ),x=0..sqrt(1-y^2)),y=0..1);evalf(%);#in rect's
```

$$4 \int_0^1 \int_0^{\sqrt{1-y^2}} \int_{\sqrt{x^2+y^2}}^1 1\, dz\, dx\, dy = 4 \int_0^1 \frac{1}{4} y^2 \ln(y^2) - \frac{1}{2} y^2 \ln(\sqrt{1-y^2}+1) + \frac{\sqrt{1-y^2}}{2}\, dy$$

$$4. \int_{0.}^{1.} \int_{0.}^{\sqrt{1. - 1.\, y^2}} \int_{\sqrt{x^2+y^2}}^{1.} 1.\, dz\, dx\, dy = 1.047197551$$

```
>
```