With the given function, we plot the graph of the function:

\[ f(x, y) = -x^2 + xy - 2y - 2x - y^2 + 4 \]

Then, we find the partial derivatives of the function at the point (x, y) and set them equal to zero:

\[ \frac{\partial f}{\partial x} = 0 \quad \text{and} \quad \frac{\partial f}{\partial y} = 0 \]

This sets up two simultaneous equations.

Warning, the name changecoords has been redefined
Warning, the assigned names <,> and <|> now have a global binding
Warning, these protected names have been redefined and unprotected: *, +, -, ., D, Vector, diff, int, limit, series

```plaintext
> restart; with(plots): with(student): with(Student[VectorCalculus]):

Warning, the name changecoords has been redefined
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> f:=(x,y)-> -x^2+x*y-2*y-2*x-y^2+4;
f := (x, y) -> -x^2 + x*y - 2*y - 2*x - y^2 + 4

> plot3d( f(x,y),x=-10 .. 10,y=-10 .. 10,axes = boxed);

> fx:=diff(f(x,y),x)=0; fy:=diff(f(x,y),y)=0;
fx := -2 x + y - 2 = 0
fy := x - 2 - 2 y = 0

# two simultaneous equation
```
\[ \text{fsolve}(-2*(2+2*y)+y-2=0); \# \text{so the solution is } (-2,-2) \text{ or another way} \]

\[ -2. \]

\[ \text{poly:} = \{ f(x,y) \}; \]
\[ \text{fsolve( poly);} \]

\[ \{ y = -2., x = -2. \} \]

\[ h := \text{diff}(f(x,y),x,x) \ast \text{diff}(f(x,y),y,y) - \text{diff}(f(x,y),x,y) \ast \text{diff}(f(x,y),x,y); \]
\[ h := 3 \]

\[ \text{diff}(f(x,y),x,x); \# \text{conclusion is a local max} \]

\[ -2 \]

\[ f:=(x,y)\to 3*x*y-y^3-x^3+4; \]
\[ f := (x, y) \to \text{VectorCalculus:-`+`}(\text{VectorCalculus:-`+`}(\text{VectorCalculus:-`*`}(x, y), \text{VectorCalculus:-`-`}(y^3)), 4) \]
plot3d( f(x,y), x=-10 .. 10, y=-10 .. 10, axes = boxed);

fx:=diff(f(x,y),x)=0; fy:=diff(f(x,y),y)=0;

fx := -3*y^3*x^2 = 0
fy := -3*x^3*y^2 = 0

# two simultaneous equation but non-linear here caes so sub y = x^2
> fsolve(x-(x^2)^2=0); #gives two solutions x=0 and x = 1 so y=0 and 1 also
0, 1.

> h:=diff(f(x,y),x,x)*diff(f(x,y),y,y)-diff(f(x,y),x,y)*diff(f(x,y),x,y);
h := 36 xy - 9
> diff(f(x,y),x,x); # conclusion is a local max
-6 x
> # so at (0,0) hessian is negative so saddle point

> # while at (1,1) hessis + and fxx is negative so a local min