Intersecting Cylinders

A standard problem for three dimensional integration is to find the volume of two intersecting cylinders. One hard part of this problem is seeing the shape of the intersection. We take the cylinders

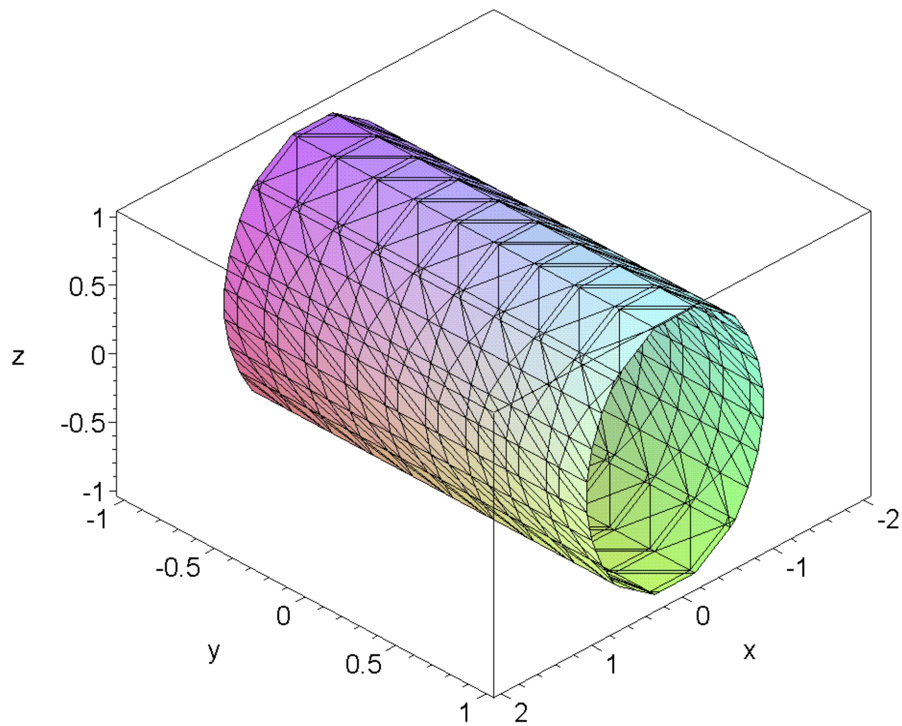
$$x^2 + y^2 = 1 \quad \text{and} \quad z^2 + x^2 = 1.$$

In an attempt to see the intersection, we first draw the two cylinders, one red and the other blue.

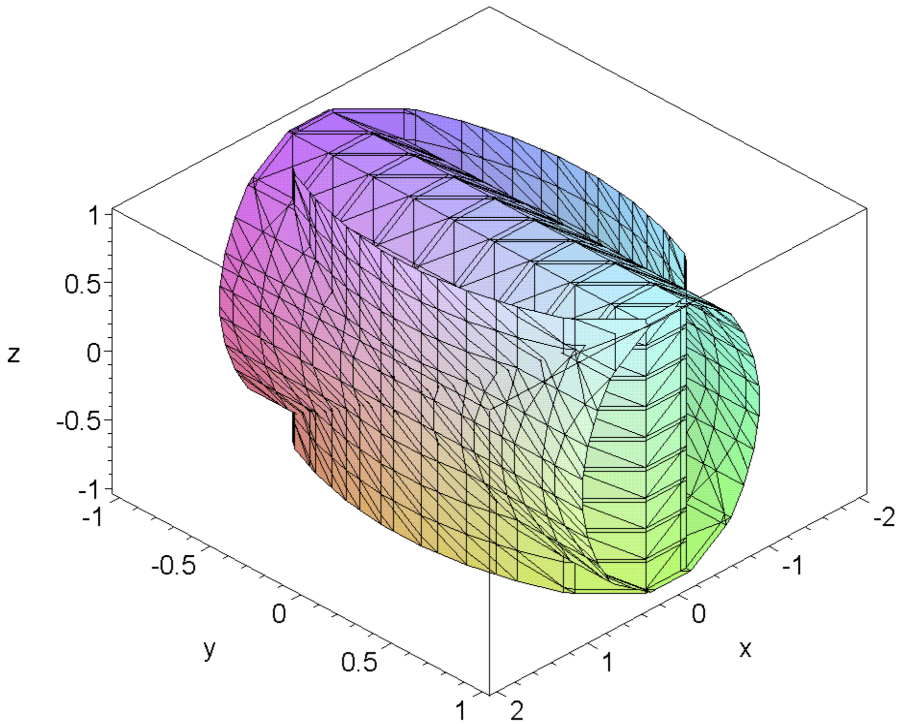
```
> restart:with(plots):with(student):
```

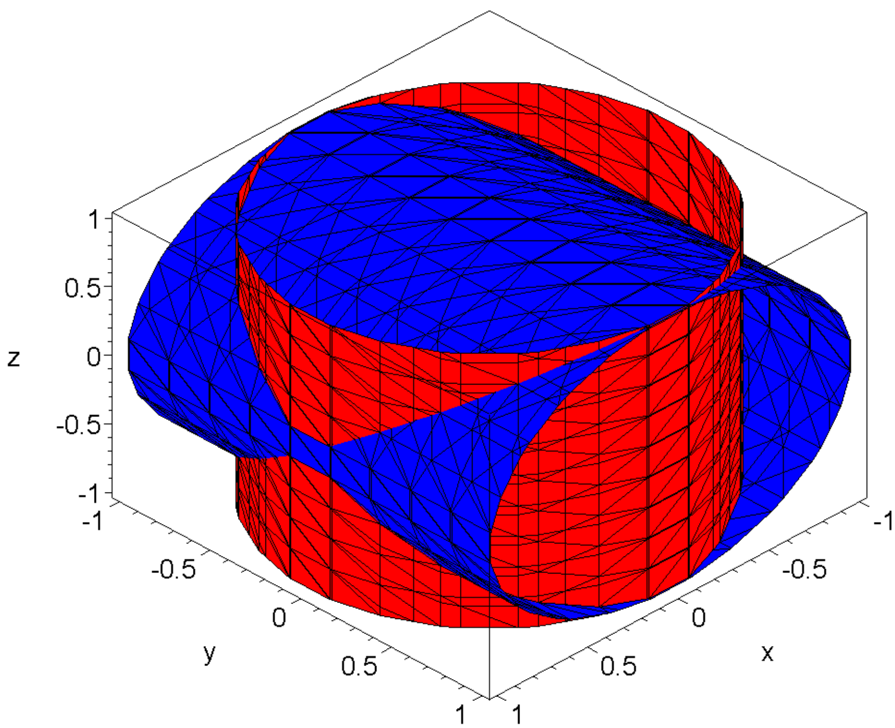
Warning, the name changecoords has been redefined

```
> implicitplot3d({ x^2 + z^2 =1},x=-2..2,y=-1..1,
  z=-1..1,axes=boxed);
```




```
> implicitplot3d({x^2 + y^2 = 1,x^2 + z^2 = 1},x=-2..2,y=-1..1,
  z=-1..1,axes=boxed);J:=implicitplot3d(x^2 + y^2 =
  1,x=-1..1,y=-1..1,
  z=-1..1,axes = boxed,color=RED):K:=implicitplot3d(x^2 + z^2 =
  1,x=-1..1,y=-1..1,
  z=-1..1,axes = boxed,color=BLUE):display3d({J,K});
```






```
> 8*Int(Int(Int( 1, z=0..sqrt(1-x^2)
```

```
),y=0..sqrt(1-x^2)),x=0..1)=8*int(int(int( 1, z=0..sqrt(1-x^2)
),y=0..sqrt(1-x^2)),x=0..1);#easy way
```

$$8\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2}} 1\,dz\,dy\,dx = \frac{16}{3}$$

```
> 8*Int(Int(Int( 1, z=0..sqrt(1-x^2)
),x=0..sqrt(1-y^2)),y=0..1)=8*Int(Int(int( 1, z=0..sqrt(1-x^2)
),x=0..sqrt(1-y^2)),y=0..1);8*Int(int(int( 1, z=0..sqrt(1-x^2)
),x=0..sqrt(1-y^2)),y=0..1);8*int(int(int( 1, z=0..sqrt(1-x^2)
),x=0..sqrt(1-y^2)),y=0..1);# a little harder
```

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$$8\int_0^1\int_0^{\sqrt{1-y^2}}\int_0^{\sqrt{1-x^2}} 1\,dz\,dx\,dy = 8\int_0^1\int_0^{\sqrt{1-y^2}} \sqrt{1-x^2}\,dx\,dy$$

$$8\int_0^1 \frac{1}{2}\sqrt{1-y^2}\,\mathrm{csgn}(y)\,y + \frac{1}{2}\arcsin(\sqrt{1-y^2})\,dy$$

$$\frac{16}{3}$$

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```
> 2*Int(Int(Int( r, z=0..sqrt(1-(r*cos(theta))^2)
),r=0..1),theta=0..2*Pi)=2*int(int(int( r,
z=0..sqrt(1-(r*cos(theta))^2) ),r=0..1),theta=0..2*Pi);evalf(%);#
cylindrical coor
```

$$2\int_0^{2\pi}\int_0^1\int_0^{\sqrt{1-r^2\cos(\theta)^2}} r\,dz\,dr\,d\theta = \frac{16}{3}$$

$$2.\int_{0.}^{6.283185308}\int_{0.}^{1.}\int_{0.}^{\sqrt{1.-1.\,r^2\cos(\theta)^2}} r\,dz\,dr\,d\theta = 5.333333333$$

```
> restart:with(plots):with(student):
```

```
Warning, the name changecoords has been redefined
```

```
> 8*Int(Int(Int( rho^2*sin(phi),
  rho=0..sqrt(1/(1-(sin(theta)*sin(phi))^2))
  ),phi=0..arccot(sin(theta))),theta=0..Pi/2)=8*int(int(int(
  rho^2*sin(phi), rho=0..sqrt(1/(1-(sin(theta)*sin(phi))^2))
  ),phi=0..arccot(sin(theta))),theta=0..Pi/2);ans1:=evalf(%);
```

$$8\int_{0}^{\frac{\pi}{2}}\int_{0}^{\operatorname{arccot}(\sin(\theta))}\int_{0}^{\sqrt{\frac{1}{1-\sin(\theta)^2\sin(\phi)^2}}}\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta=\frac{8}{3}$$

$$ans1 := 2.666666666 = 2.666666667$$

```
> 8*Int(Int(Int( rho^2*sin(phi),
  rho=0..csc(phi)),phi=arccot(sin(theta))..Pi/2),theta=0..2*Pi);

>

> 8*int(int(int( rho^2*sin(phi),
  rho=0..csc(phi)),phi=arccot(sin(theta))..Pi/2),theta=0..Pi/2);ans2
  :=evalf(%);
```

$$8\int_{0}^{2\pi}\int_{\operatorname{arccot}(\sin(\theta))}^{\frac{\pi}{2}}\int_{0}^{\csc(\phi)}\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta$$

```
Warning, unable to determine if Pi*_Z17 is between arccot(sin(theta)) and 1/2*Pi;
try to use assumptions or set _EnvAllSolutions to true
```

$$8\int_{0}^{\frac{\pi}{2}}\int_{\operatorname{arccot}(\sin(\theta))}^{\frac{\pi}{2}}\frac{1}{3}\sin(\phi)\csc(\phi)^3\,d\phi\,d\theta$$

```
Warning, unable to determine if Pi*_Z37 is between arccot(sin(theta)) and 1/2*Pi;
try to use assumptions or set _EnvAllSolutions to true
```

$$ans2 := 2.666666666$$

$$8\int_{0}^{\frac{\pi}{2}}\int_{\operatorname{arccot}(\sin(\theta))}^{\frac{\pi}{2}}\frac{1}{3}\sin(\phi)\csc(\phi)^3\,d\phi\,d\theta$$

$$ans2 := 2.666666666$$

```
> evalf(ans1+ans2);
```

$$5.333333332 = 5.333333333$$

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