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Math. 2673 Sample Final Exam Spring, 2007

I

1. If $u = x^y$, show that

$$\frac{x}{y}\frac{\partial u}{\partial x} + \frac{1}{\ln x}\frac{\partial u}{\partial y} = 2u$$

2. Find the maximum rate of change of $f(x, y) = x^2y + \sqrt{(y)}$ at the point $(2, 1)$. In which direction does this max occur?

3. Find the local maximum and minimum values and saddle points of the given function.

$f(x, y) = 3xy - x^2y - xy^2$

4. If $z = \cos(xy) + y\cos(x)$, where $x = u^2 + v$ and $y = u - v^2$, use the Chain Rule to find $\frac{\partial z}{\partial u}$ and $\frac{\partial^2 z}{\partial u^2}$

II

1.) Graph the region and find the volume inside both the sphere $x^2 + y^2 + z^2 = 4$ and exterior to the sphere $x^2 + y^2 + z^2 = 1$.

2.) Graph the region and find the volume bounded by the cone whose angle in spherical coordinates is given by $\phi = \frac{\pi}{3}$ and a sphere of radius 2.

III

3.) Verify Green's Theorem for $\int_C (x - y)dx + (x + y)dy$ where C is in the counterclockwise direction about the triangle with vertices at $(0, 0), (0, 1), (1, 0),)$

4.) Integrate $f(x, y) = x + y$ over the curve formed by C_1 and C_2

C_1 :line seqment between $(0, 0) to (1, 1)$ and

C_2 : line seqment between $(1, 1) to (2, 3)$

5.) Let $F(x,y) = y\imath + x\jmath$ be a continuous velocity field and consider the closed path γ given by the $r(t)$ which is the union of the arch $r_1(t) = \cos(t)\imath + \sin(t)\jmath, 0 \leq t \leq \pi$

followed by the line segment ($r_2(t)$) $(-1,0)$, and $(1,0)$

Find the $\int_\gamma F(x,y)\dot{dr}$

III.)

6.) Find $f(x,y,z)$ such that $\nabla f = F$ for each of the following:

a.) $F(x,y,z) = (yz, xz, xy)$

7.) For $F(x,y,z) = \sin(x)\imath + \sin(y)\jmath + xz^2k$

a.) Find $\nabla(\nabla \cdot F)$

b.) Find $\nabla \times (\nabla \times F)$

IV

2.) Find the volume of the solid in the first octant bounded by the plane $x + 2y + z = 4$.

3.) Find the volume inside the paraboloid $z = (x^2 + y^2)$ and below the plane $z = 1$.

5.) Find the volume inside both the sphere $x^2 + y^2 + z^2 = 2$ and the paraboloid $z = x^2 + y^2$.

8. If Ω =the triangle whose vertices are:$(0,0).(1,0),(1,1)$ and γ the boundary of Ω.

Set $M(x,y) = -ye^x$ and $N(x,y) = xe^y$

Verify Green's Theorem

11.) Evaluate $\int_C (x-y)dx + (x+y)dy$ where C is the unit square with a vertices at $(0,0), (0,1), (1,0), (1,1)$