2. If \( u = x^y \), show that
\[
\frac{x \partial u}{y \partial x} + \frac{1}{\ln x} \frac{\partial u}{\partial y} = 2u
\]

3. Find the maximum rate of change of \( f(x, y) = x^2y + \sqrt{y} \) at the point \((2,1)\). In which direction does this max occur?

4. Find the local maximum and minimum values and saddle points of the given function.
\[
f(x, y) = 3xy - x^2y - xy^2
\]

5. If \( z = \cos(xy) + y \cos(x) \), where \( x = u^2 + v \) and \( y = u - v^2 \), use the Chain Rule to find \( \frac{\partial z}{\partial u} \) and \( \frac{\partial^2 z}{\partial u^2} \)

1.) Graph the region and find the volume inside both the sphere \( x^2 + y^2 + z^2 = 4 \) and exterior to the sphere \( x^2 + y^2 + z^2 = 1 \).

2.) Graph the region and find the volume bounded by the cone whose angle in spherical coordinates is given by \( \phi = \frac{\pi}{4} \) and a sphere of radius 2.

II

3.) Verify Green’s Theorem for \( \int_C (x - y)dx + (x + y)dy \) where \( C \) is in the counterclockwise direction about the triangle with vertices at \((0,0), (0,1), (1,0)\).

4.) Integrate \( f(x, y) = x + y \) over the curve formed by \( C_1 \) and \( C_2 \)

\( C_1 : \) line segment between \((0,0)\)to\((1,1)\) and

\( C_2 : \) line segment between \((1,1)\)to\((2,3)\)

5.) Let \( F(x, y) = yi + xj \) be a continuous velocity field and consider the closed path \( \gamma \) given by the \( r(t) \) which is the union of the arch \( r_1(t) = \cos(t)i + \sin(t)j, 0 \leq t \leq \pi \)

followed by the line segment \( r_2(t) \) \((-1,0), \) and \((1,0)\)

Find the \( \int_\gamma F(x, y)dr \)
III.)

6.) Find \( f(x, y, z) \) such that \( \nabla f = F \) for each of the following:

a.) \( F(x, y, z) = (yz, xz, xy) \)

7.) For \( F(x, y, z) = \sin(x)i + \sin(y)j + xz^2k \)

a.) Find \( \nabla (\nabla \cdot F) \)

b.) Find \( \nabla \times (\nabla \times F) \)

2.) Find the volume of the solid in the first octant bounded by the plane \( x + 2y + z = 4 \).

3.) Find the volume inside the paraboloid \( z = (x^2 + y^2) \) and below the plane \( z = 1 \).

5.) Find the volume inside both the sphere \( x^2 + y^2 + z^2 = 2 \) and the paraboloid \( z = x^2 + y^2 \).

7.) The solid in the first octant volume bounded by the cylinder \( z^2 + y^2 = 9 \) and \( z = x^2 + 3y^2 \)

8. If \( \Omega \) = the triangle whose vertices are: \((0, 0), (1, 0), (1, 1)\) and \( \gamma \) the boundary of \( \Omega \).

Set \( M(x, y) = -ye^x \) and \( N(x, y) = xe^y \)

Verify Green’s Theorem

11.) Evaluate \( \int_C (x-y)dx + (x+y)dy \) where \( C \) is the unit square with vertices at \((0, 0), (0, 1), (1, 0), (1, 1)\)