MATH 2673 | Calculus III: Exam 2 | NAME | 3/19/2009

1 Gragient, Tangent Plane and Aprroximation

1. For $f(x, y) = x * \ln(4 * y + x^2)$.
   (a) Find the tangent plane at the point $P = (1, 0)$
   (b) Use this tangent plane to approximate $(1.1) * \ln[4 * (0.05) + (1.1)^2]$.

2. Let $f(x, y) = (x^2 + y^2)$
   (a) Find the rate of change of f as we move from the point $P = (1, 4)$ in the direction of the point $Q = (4, 8)$. (not trying to trick you but you need the vector.)
   (b) In what direction is the maximum rate of change?
   (c) What is that maximum rate of change ?

2 Partial Differentiation and the Chain rule

1. use the **(ONLY)** Chain Rule to find $\frac{dw}{dt}$ at $t = 1$.

   where $w = y * z^2 - x$ where and $x = t^2$, $y = 2t - 3$ and $z = 1 - t$.

2. use the **(ONLY)** Chain Rule to find $\frac{d^2w}{dt^2}$ at $t = \pi/4$.

   where $w = x^3$ where and $x = (\sin(t))^2$.

3. use **(ONLY)** the Chain Rule to find $\frac{\partial w}{\partial x}$ where $w = v^3 - 2 * u$ and $u = 4 * x * y - x + 1$ and $v = y + x * y^2$.

4. Show $f(x, y, z) = 2 * z^3 - 3(x^2 + y^2) * z$ satisfies $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$

3 classify critical points

1. Locate and classify the critical points of the function

   $f(x, y) = x^2 + y^3 - 6xy.$

2. Let S be the closed region bounded by the triangle $S = (x, y) : 0 \leqslant x \leqslant 1, 0 \leqslant y \leqslant 1, y \leqslant x$ See board! Determine the absolute min and max on S of the function

   $f(x, y) = x^2 + 4xy - y^2 - 5x$

3. Use the Lagrange Multiplier Method to determine the extreme values of the function $f(x, y, z) = xyz$ subject to the constraint $x^2 + y^2 + z^2 = 1$ (Hint: think how we did the problem in the book on tuesday! The answer is $x^2 = y^2 = z^2, ...$)