MATH 2673 | Calculus III: Sample Final Exam | 4/25/2009

1 Test 2 Material

1. For $f(x, y) = x * \ln(4 * y + x^2)$.
   (a) Find the tangent plane at the point $P = (1, 0)$
2. Let $f(x, y) = (x^2 + y^2)$
   (a) Find the rate of change of f as we move from the point $P = (1, 4)$ in the direction of the point $Q = (4, 8)$. (not trying to trick you but you need the vector.)
   (b) In what direction is the maximum rate of change?
   (c) What is that maximum rate of change ?
3. Use the **(ONLY)** Chain Rule to find $\frac{dw}{dt}$ at $t = 1$.
   where $w = y * z^2 - x$ where and $x = t^2$, $y = 2t - 3$ and $z = 1 - t$.
   where $w = x^3$ where and $x = (\sin(t))^2$.
4. use **(ONLY)** the Chain Rule to find $\frac{\partial w}{\partial x}$ where $w = v^3 - 2 * u$ and $u = 4 * x * y - x + 1$ and $v = y + x * y^2$.
5. Show $f(x, y, z) = 2 * z^3 - 3(x^2 + y^2) * z$ satisfies $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$
6. Locate and classify the critical points of the function
   $f(x, y) = x^2 + y^3 - 6xy$.

2 Test 3 Material

1. Find the volume bounded by the two surfaces $z = 8 - x^2 - y^2$ and $z = x^2 + 3y^2$
2. Find the volume inside both the sphere $x^2 + y^2 + z^2 = 9$ and the cylinder $x^2 + y^2 = 1$.
3. The volume in the first octant bounded by planes $x = 1$, $y = 2$, and $z = 3$
4. The volume in the first octant of the cylinder of radius 2 with height 2.

5. The volume in the first octant of the sphere of radius 2 .
6. The volume bounded by the cone whose angle in spherical coordiantes is given by $\phi = \frac{\pi}{4}$ and a sphere of radius2.

3 New Material

1. Integrate $f(x, y) = x^3/y$ over the curve formed by C_1 and C_2
   $C_1 : y = x^2/2, 0 \leq x \leq 2$ and
   C_2 : line seqment between $(2, 2) and (4, 3)$
2. Let $F(x, y) = -y\imath + x\jmath$ be a continuous velocity field and consider the closed path γ given by the arch $r_1(t) = 3\cos(t)\imath + 3\sin(t)\jmath, 0 \leq t \leq \pi$ followed by the line segment ($r_2(t)$) $(-3, 0)$, and $(3, 0)$
3. Find $f(x, y)$ such that $\nabla f = F$ for each of the following:
   (a) $F(x, y, z) = (yz, xz, xy)$
   (b) $F(x, y, z) = ((\sec(x))^2, z, y + 2z)$
4. For $F(x, y, z) = y\sin(z)\imath + x\sin(z)\jmath + xy\cos(z)k$
   (a) Find $\nabla \cdot F$
   (b) Find $\nabla \times F$
   (c) Find $\nabla(\nabla \cdot F)$
   (d) Find $\nabla \cdot (\nabla \times F)$
5. If $\Omega = (x, y) : 0 \leq x \leq +1, 0 \leq y \leq +1$ and γ the boundary of Ω.
   Use Green's theorem to evaluate the following:
   $\int_\gamma -ye^x dx + xe^y dy$
6. Suppose $F(x, y) = (3x^2y, x^3 + 2y)$
   (a) Find $f(x, y)$ such that $\nabla f = F$.
   (b) Use this to evaluate $\int_a^b F \cdot dr$ where $a = (1, 1)$ & $b = (2, 3)$.
   (c) Use this to evaluate $\int_\Gamma F \cdot dr$ where Γ is the circle of radius a.