```
> restart:with(plots):with(linalg):
  with(student):Digits := 4:
```

First Test, Calculus 2673, 2/12/2009

1. i

```
> u:= vector([6,3,2]);v:=vector([1,-2,-2]);
```

$$u := \begin{bmatrix} 6 & 3 & 2 \end{bmatrix}$$

$$v := \begin{bmatrix} 1 & -2 & -2 \end{bmatrix} \quad (1)$$

a

```
> theta := evalf(arccos((dotprod(u,v)/(norm(v,2)* norm(u,2))))/Pi*
  180);
```

$$\theta := 101.0 \quad (2)$$

this says that the angle is an obtuse angle; but no fear, we want the u1 to be parallel only. so turn v in the other direction

b

```
>
> v := vector([-1,+2,2]);
```

$$v := \begin{bmatrix} -1 & 2 & 2 \end{bmatrix} \quad (3)$$

```
> theta := evalf(arccos((dotprod(u,v)/(norm(v,2)*norm(u,2))))/Pi*180);
```

$$\theta := 79.00 \quad (4)$$

```
> u1:=  evalm ((dotprod(u,v)/(norm(v,2)))*(v/norm(v,2) ));
```

$$u1 := \begin{bmatrix} -\frac{4}{9} & \frac{8}{9} & \frac{8}{9} \end{bmatrix} \quad (5)$$

```
> u2:=evalm(u-u1);
```

$$u2 := \begin{bmatrix} \frac{58}{9} & \frac{19}{9} & \frac{10}{9} \end{bmatrix} \quad (6)$$

show u1 and u2 are perpendicular

```
> dotprod(u1,u2);
```

$$0 \quad (7)$$

```
>  crossprod(u,v);
```

$$\begin{bmatrix} 2 & -14 & 15 \end{bmatrix} \quad (8)$$

2. Let P= (-3,0,7) and N is the vector <5,2,-1>

a. Find the equation of the plane that contains P and N is thenormal

```
> P:= vector([-3,0,7]);N:= vector([5,2,-1]);
```

$$P := \begin{bmatrix} -3 & 0 & 7 \end{bmatrix}$$

$$N := \begin{bmatrix} 5 & 2 & -1 \end{bmatrix} \quad (9)$$

```
> R:=vector([x,y,z]);# template to pick up the varaibles
```

$$R := \begin{bmatrix} x & y & z \end{bmatrix} \quad (10)$$

```
> u:=dotprod(R,N);k :=(dotprod(N,P)); # direction and konstant
```

$$u := 5x + 2y - z$$

$$k := -22 \quad (11)$$

```
> u = k; # equation of the  plane
```

$$5x + 2y - z = -22 \quad (12)$$

b the line that contains P and is parralel to N

.

```
> P:= vector([-3,0,7]);N:= vector([5,2,-1]);
```

$$P := \begin{bmatrix} -3 & 0 & 7 \end{bmatrix}$$

$$N := \begin{bmatrix} 5 & 2 & -1 \end{bmatrix} \quad (13)$$

```
> L:=evalm(t*N +P);# template to pick up the varaibles
```

$$L := \begin{bmatrix} 5t - 3 & 2t & -t + 7 \end{bmatrix} \quad (14)$$

```
> #c line per to N, find a vector say v =<0,1,2>
> v:= vector([0,1,2]);dotprod(N,v);l:=evalm(P+t*v);
```

$$v := \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$$

$$0$$

$$l := \begin{bmatrix} -3 & t & 7 + 2t \end{bmatrix} \quad (15)$$

3.

```
>
>
> x := t → t·sin(t) + cos(t);
```

$$x := t \rightarrow t \sin(t) + \cos(t) \quad (16)$$

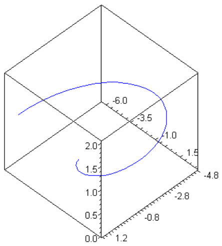
```
> y :=  t → sin(t) - t·cos(t);
```

$$y := t \rightarrow \sin(t) - t\cos(t) \quad (17)$$

```
> z := t → 1;
```

$$z := t \rightarrow 1 \quad (18)$$

```
> spacecurve( [x(t), y(t), z(t) ], t = 0 .. 2 * Pi, color = blue, axes = boxed);
```




```
> r := t → vector([t·sin(t) + cos(t), sin(t) − t·cos(t), 1]);
  rt := t → vector([t*cos(t), t·sin(t), 0]); rtt := t → vector([cos(t) − t·sin(t), sin(t) + t
      ·cos(t), 0]);
```

$$r := t \to linalg\text{:-}vector([t\sin(t) + \cos(t), \sin(t) - t\cos(t), 1])$$

$$rt := t \to linalg\text{:-}vector([t\cos(t), t\sin(t), 0])$$

$$rtt := t \to linalg\text{:-}vector([\cos(t) - t\sin(t), \sin(t) + t\cos(t), 0]) \quad \textbf{(19)}$$

```
> v := t ; # velocity is t
```

$$v := t \quad \textbf{(20)}$$

```
> T := t → vector([cos(t), sin(t), 0]);  # a unit vector
```

$$T := t \to linalg\text{:-}vector([\cos(t), \sin(t), 0]) \quad \textbf{(21)}$$

```
>
```

```
>  N := t → vector ([−sin(t), cos(t), 0]);
```

$$N := t \to linalg\text{:-}vector\ [-\sin(t), \cos(t), 0] \quad \textbf{(22)}$$

```
>
```

```
>
>
> # EX 4

> P:=[1,1,3];
```

$$P := [1, 1, 3] \quad (23)$$

```
> #a l:= <1,-1,2> and find a point on the line say P0 =(1,3,0)
  form the  Po to P vector
> Po:=vector([1,3,0]); PotoP:= evalm(P-Po);
```

$$Po := \begin{bmatrix} 1 & 3 & 0 \end{bmatrix}$$

$$PotoP := \begin{bmatrix} 0 & -2 & 3 \end{bmatrix} \quad (24)$$

```
>  crossprod(l,PotoP);l:=vector([1,-1,2]); # the line vector
```

$$\begin{bmatrix} 7\,t+14 & 9 & 6 \end{bmatrix}$$

$$l := \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} \quad (25)$$

```
> distanc := eval(norm( crossprod(l,PotoP),2)/( norm(l,2)));
  simplify(%);
```

$$distanc := \frac{1}{6}\sqrt{14}\sqrt{6}$$

$$\frac{1}{3}\sqrt{7}\sqrt{3} \quad (26)$$

b

```
> #a N:= <3,2,6> and find a point on the plane  say P0 =(0,0,1)
  form the  Po to P vector
> Po:=vector([0,0,1]); PotoP:= evalm(P-Po);N:= vector([3,2,6]) ;
```

$$Po := \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$PotoP := \begin{bmatrix} 1 & 1 & 2 \end{bmatrix}$$

$$N := \begin{bmatrix} 3 & 2 & 6 \end{bmatrix} \quad (27)$$

```
>
> distanc := eval(( dotprod(N,PotoP),2)/( norm(N,2)));simplify(%);
```

$$distanc := \frac{17}{7}, \frac{2}{7}$$

$$\frac{17}{7} \quad (28)$$

```
>
```

```
> #Ex 5 it is obvious how to create the vectors
> u:= vector([3,2,1]);v:=vector([1,1,1]); # so the N normal to the
  plane is given by
```

$$u := \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$$

$$v := \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \quad (29)$$

```
> N:= crossprod(u,v);
```

$$N := \begin{bmatrix} 1 & -2 & 1 \end{bmatrix} \quad (30)$$

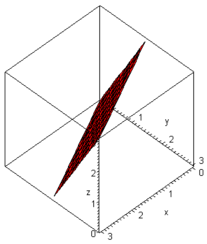
```
>
> R:=vector([x,y,z]);# template to pick up the varaibles
```

$$R := \begin{bmatrix} x & y & z \end{bmatrix} \quad (31)$$

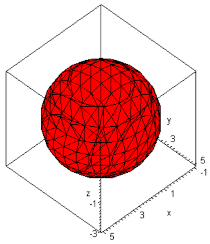
```
> PLane:=dotprod(R,N)-(dotprod(N,P)); # direction and konstant
```

$$PLane := x - 2y + z - 2 \quad (32)$$

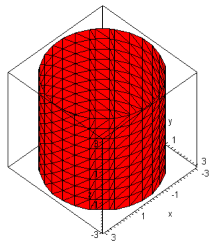
> $implicitplot3d(\ x - 2 \cdot y\ + z = 0, x = 0 .. 3, y = 0 .. 3, z = 0 ..3, color = red, axes = boxed);$



```
> # Ex 6
> implicitplot3d( x^2 + y^2  + z^2 - 4·x - 4·y = 0, x = -1 .. 5, y =-1 .. 5, z =-3 ..3, color = red,
      axes = boxed);
```




```
> implicitplot3d( x^2+y^2-9=0, x= -3 .. 3,y=-3 .. 3, z=-3 ..3,
  color = red, axes = boxed);
```




```
> v0 := t -> vector([44/sqrt(2),44/sqrt(2)]); # initial velocity
```

$$v0 := t \rightarrow linalg\text{:-}vector\left(\left[\frac{44}{\sqrt{2}}, \frac{44}{\sqrt{2}}\right]\right) \quad \textbf{(33)}$$

```
>  a: t ->  vector([0,-32]);
```

$$t \rightarrow linalg\text{:-}vector([0, -32]) \quad \textbf{(34)}$$

```
>
> v := t -> vector([44/sqrt(2),-32*t+44/sqrt(2)]) ;
```

$$v := t \rightarrow linalg\text{:-}vector\left(\left[\frac{44}{\sqrt{2}}, -32\, t + \frac{44}{\sqrt{2}}\right]\right) \quad \textbf{(35)}$$

```
>  r0 := t -> vector([0,13/2]); # initial height
```

$$r0 := t \rightarrow linalg\text{:-}vector\left(\left[0, \frac{13}{2}\right]\right) \quad \textbf{(36)}$$

```
>
```

$$> \; r := t \rightarrow vector\left(\left[\frac{44}{\mathrm{sqrt}(2)} \cdot t, -16 \cdot t^2 + \frac{44}{\mathrm{sqrt}(2)} \cdot t + \frac{13}{2}\right]\right);$$

(37)

$$r := t \to linalg\text{:-}vector\left(\left[\frac{44\,t}{\sqrt{2}},\ -16\,t^2 + \frac{44\,t}{\sqrt{2}} + \frac{13}{2}\right]\right) \quad (37)$$

> # a find max height , occurs when j component of velocity is zero

> $solve\left(-32\cdot t + \frac{44}{\mathrm{sqrt}(2)} = 0, t\right)$;# then place that into the j comp of the r(t) equation

$$\frac{11}{16}\sqrt{2} \quad (38)$$

> $tmax := \%;$

$$tmax := \frac{11}{16}\sqrt{2} \quad (39)$$

> $eval(r(tmax))$; # the ans is the j comp

$$\left[\frac{121}{4} \quad \frac{173}{8}\right] \quad (40)$$

> # b   find the ranger , time when it hits the ground

> $solve\left(-16\cdot t^2 + \frac{44}{\mathrm{sqrt}(2)}\cdot t + \frac{13}{2} = 0, t\right);$

select the positive route then place that into the I comp of the r(t) equation

$$\frac{11}{16}\sqrt{2} - \frac{1}{16}\sqrt{346},\ \frac{11}{16}\sqrt{2} + \frac{1}{16}\sqrt{346} \quad (41)$$

> $trange := \frac{11}{16}\cdot\mathrm{sqrt}(2) + \frac{\mathrm{sqrt}(346)}{16};$

$$trange := \frac{11}{16}\sqrt{2} + \frac{1}{16}\sqrt{346} \quad (42)$$

> $evalf(r(trange))$; # range is 66.37

$$[66.37 \quad 0.010] \quad (43)$$

> # c nedd the norm of the velocity equation at t = trange

> $evalf(v(trange));$

$$[31.11 \quad -37.20] \quad (44)$$

> $norm(\%, 2)$; # nowm of the previous answer

$$48.50 \quad (45)$$

>