

```
> restart:with(student):with(plots);
Warning, the name changecoords has been redefined
```

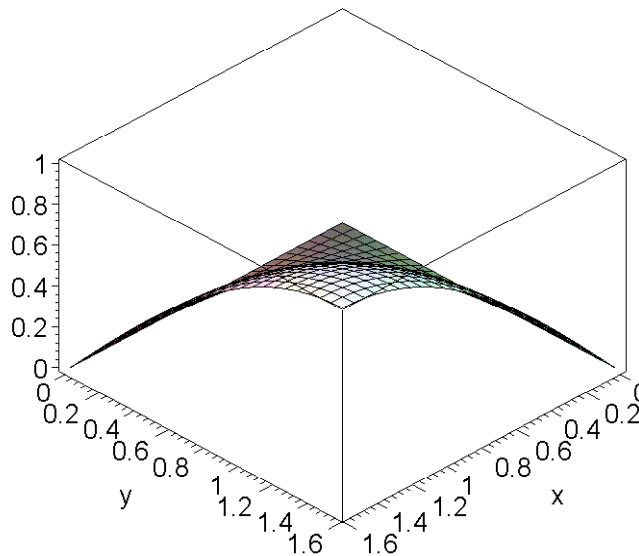
```
[animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot,
contourplot3d, coordplot, coordplot3d, cylinderplot, densityplot, display, display3d, fieldplot, fieldplot3d, gradplot, gradplot3d,
implicitplot, implicitplot3d, inequal, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot,
odeplot, pareto, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, replot,
rootlocus, semilogplot, setoptions, setoptions3d, spacecurve, sparsematrixplot, sphereplot, surfdata, textplot, textplot3d, tubeplot]
```

```
>
>
>
```

```
> g:= (x,y)-> sin(x)*sin(y);
```

```
g := (x, y) → sin(x) sin(y)
```

```
> plot3d(sin(x)*sin(y), x=0..Pi/2, y=0..Pi/2, axes=boxed);
```



```
> N:=4;
```

```
N := 4
```

```
> delx:=Pi/2/N;dely:=Pi/2/N;
```

```
>
```

```
delx := 1/8 pi
```

```
dely := 1/8 pi
```

```
>
```

```
> Sum(Sum('g((i-1)*delx+delx/2, (j-1)*dely+dely/2)*delx*dely',
'i'=1..N), 'j'=1..N)=sum(sum('g(evalf((i-1)*delx+delx/2), evalf((j-1)*dely+dely/2))*delx*d
ely'), 'i'=1..N), 'j'=1..N);
```

$$\sum_{j=1}^4 \left(\sum_{i=1}^4 g \left((i-1) \operatorname{delx} + \frac{1}{2} \operatorname{delx}, (j-1) \operatorname{dely} + \frac{1}{2} \operatorname{dely} \right) \operatorname{delx} \operatorname{dely} \right) = 1.012950747$$

```
> Sum(Sum('g((i-1)*delx+delx/2, (j-1)*dely+dely/2)*delx*dely',
'i'=1..N), 'j'=1..N)=(sum(sum(evalf('g((i-1)*delx+delx/2, (j-1)*dely+dely/2)*delx*dely'),
'i'=1..N), 'j'=1..N));
```

$$\sum_{j=1}^4 \left(\sum_{i=1}^4 g \left((i-1) \operatorname{delx} + \frac{1}{2} \operatorname{delx}, (j-1) \operatorname{dely} + \frac{1}{2} \operatorname{dely} \right) \operatorname{delx} \operatorname{dely} \right) = \frac{1}{64} \sin \left(\frac{1}{16} \pi \right)^2 \pi^2 + \frac{1}{32} \sin \left(\frac{3}{16} \pi \right) \sin \left(\frac{1}{16} \pi \right) \pi^2$$

$$\begin{aligned}
& + \frac{1}{32} \sin\left(\frac{5}{16} \pi\right) \sin\left(\frac{1}{16} \pi\right) \pi^2 + \frac{1}{32} \sin\left(\frac{7}{16} \pi\right) \sin\left(\frac{1}{16} \pi\right) \pi^2 + \frac{1}{64} \sin\left(\frac{3}{16} \pi\right)^2 \pi^2 + \frac{1}{32} \sin\left(\frac{5}{16} \pi\right) \sin\left(\frac{3}{16} \pi\right) \pi^2 \\
& + \frac{1}{32} \sin\left(\frac{7}{16} \pi\right) \sin\left(\frac{3}{16} \pi\right) \pi^2 + \frac{1}{64} \sin\left(\frac{5}{16} \pi\right)^2 \pi^2 + \frac{1}{32} \sin\left(\frac{7}{16} \pi\right) \sin\left(\frac{5}{16} \pi\right) \pi^2 + \frac{1}{64} \sin\left(\frac{7}{16} \pi\right)^2 \pi^2
\end{aligned}$$

```
> Int( Int(g(x,y), x=0 ..Pi/2), y=0..Pi/2) =int( int(g(x,y), x=0 ..Pi/2), y=0..Pi/2) ;
```

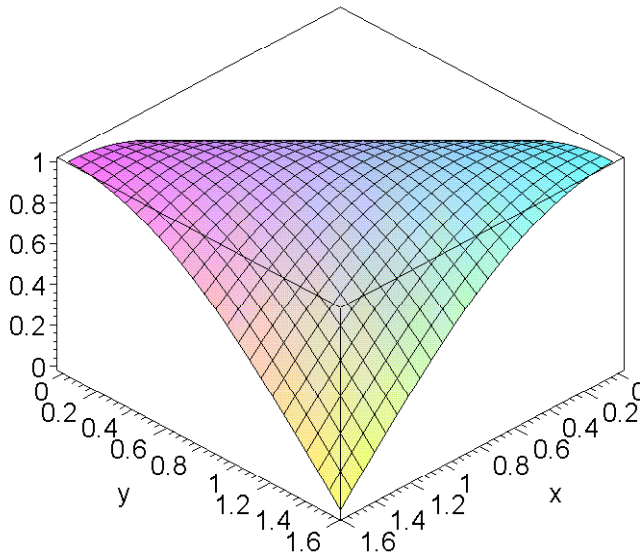
$$\int_0^{1/2\pi} \int_0^{1/2\pi} \sin(x) \sin(y) dx dy = 1$$

```
>
```

```
> G:= (x,y)-> sin(x+y);
```

$$G := (x, y) \rightarrow \sin(x + y)$$

```
> plot3d(sin(x+y), x=0..Pi/2, y=0..Pi/2, axes=boxed);
```



```
> N:=4;
```

$$N := 4$$

```
> delx:=Pi/2/N;dely:=Pi/2/N;
```

```
>
```

$$delx := \frac{1}{8} \pi$$

$$dely := \frac{1}{8} \pi$$

```
>
```

```
> Sum(Sum('G((i-1)*delx+delx/2, (j-1)*dely+dely/2)*delx*dely',
'i'=1..N), 'j'=1..N) = sum(sum('G(evalf((i-1)*delx+delx/2), evalf((j-1)*dely+dely/2))*delx*dely',
'i'=1..N), 'j'=1..N);
```

$$\sum_{j=1}^4 \left(\sum_{i=1}^4 G\left((i-1) delx + \frac{1}{2} delx, (j-1) dely + \frac{1}{2} dely\right) delx dely \right) = 2.025901494$$

```
> Sum(Sum('G((i-1)*delx+delx/2, (j-1)*dely+dely/2)*delx*dely',
'i'=1..N), 'j'=1..N) = (sum(sum(evalf('G((i-1)*delx+delx/2, (j-1)*dely+dely/2)*delx*dely',
'i'=1..N), 'j'=1..N));
```

$$\sum_{j=1}^4 \left(\sum_{i=1}^4 G\left((i-1) delx + \frac{1}{2} delx, (j-1) dely + \frac{1}{2} dely\right) delx dely \right) =$$

$$\frac{1}{64} \sin\left(\frac{1}{8}\pi\right)\pi^2 + \frac{1}{32}\sqrt{2}\pi^2 + \frac{5}{64} \sin\left(\frac{3}{8}\pi\right)\pi^2 + \frac{1}{16}\pi^2 + \frac{1}{64} \cos\left(\frac{1}{8}\pi\right)\pi^2 + \frac{1}{64} \cos\left(\frac{3}{8}\pi\right)\pi^2$$

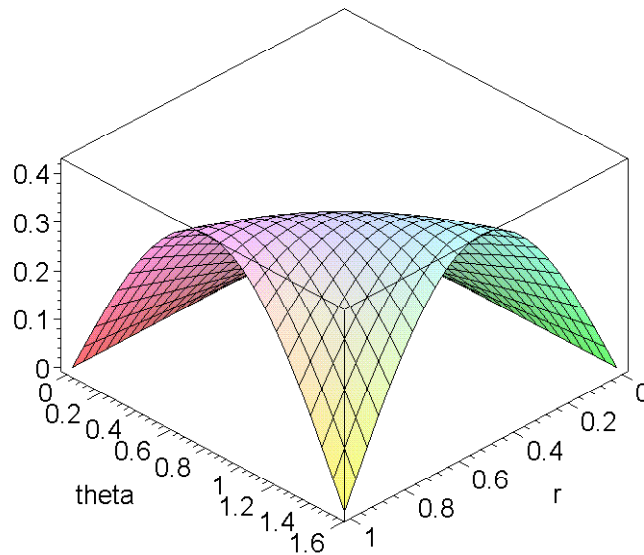
```
> Int( Int(G(x,y), x=0 ..Pi/2), y=0..Pi/2) =int( int(G(x,y), x=0 ..Pi/2), y=0..Pi/2) ;
```

$$\int_0^{1/2\pi} \int_0^{1/2\pi} \sin(x+y) dx dy = 2$$

```
> g:= (x,y)-> sin(x)*sin(y);x:=r*cos(theta);y:=r*sin(theta);
```

```
> plot3d(sin(cos(r*theta))*sin(sin(r*theta)), r=0 ..1, theta=0..Pi/2, axes=boxed);
#rect coord
```

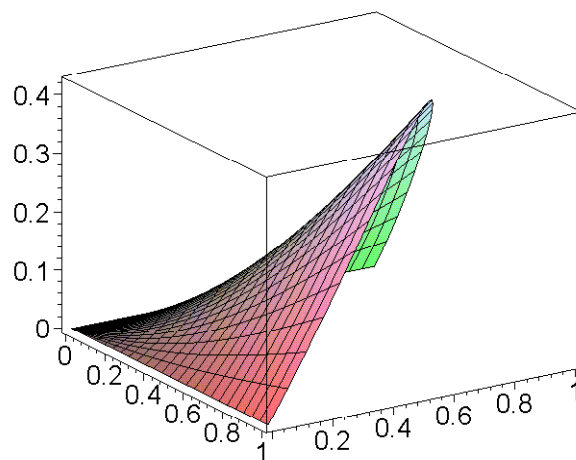
$$\begin{aligned} g &:= (x, y) \rightarrow \sin(x) \sin(y) \\ x &:= r \cos(\theta) \\ y &:= r \sin(\theta) \end{aligned}$$



```
> addcoords(z_cylindrical, [z, r, theta], [r*cos(theta), r*sin(theta), z]);
```

```
> plot3d(sin(r*cos(theta))*sin(r*sin(theta)), r=0..1, theta=0..Pi/2, coords=z_cylindrical, \
title=`cylindrical`, axes=BOXED);
```

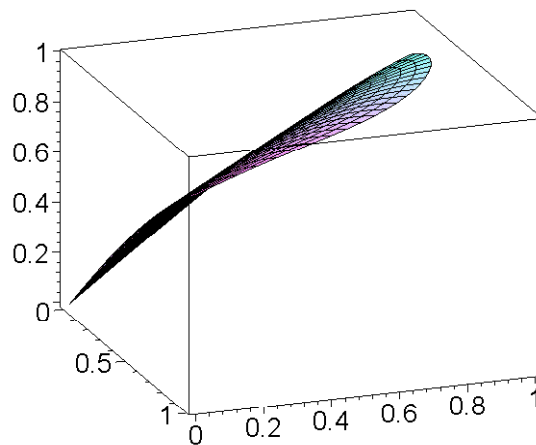
cylindrical



```
> plot3d(sin(r*cos(theta)+r*sin(theta)), r=0..1, theta=0..Pi/2, coords=z_cylindrical, \
```

```
title=`cylindrical`,axes=BOXED);
```

cylindrical



```
> N:=10;
```

$N := 10$

```
> delr:=1/N;deltheta:=Pi/2/N;
```

```
> g:= (r,theta)-> sin(r*cos(theta))*sin(r*sin(theta));
```

$delr := \frac{1}{10}$

$deltheta := \frac{1}{20} \pi$

$g := (r, \theta) \rightarrow \sin(r \cos(\theta)) \sin(r \sin(\theta))$

```
>
```

```
> Sum(Sum('g((i-1)*delr+delr/2,(j-1)*deltheta+deltheta/2)*delr*deltheta*((i-1)*delr+delr/2)',
'i'=1..N),'j'=1..N)=evalf(sum(sum('g((i-1)*delr+delr/2,(j-1)*deltheta+deltheta/2)*delr*deltheta*((i-1)*delr+delr/2)',
'i'=1..N),'j'=1..N));
```

$$\sum_{j=1}^{10} \left(\sum_{i=1}^{10} g \left((i-1) delr + \frac{1}{2} delr, (j-1) deltheta + \frac{1}{2} deltheta \right) delr deltheta \left((i-1) delr + \frac{1}{2} delr \right) \right) = .1117237310$$

```
>
```

```
> Int(Int(r*g(r,theta),theta=0..Pi/2),r=0..1)=int(int(r*g(r,theta),theta=0..Pi/2),r=0..1);
```

Warning, computation interrupted

```
> N:=10;
```

$N := 10$

```
> delr:=1/N;deltheta:=Pi/2/N;
```

```
> G:= (r,theta)-> sin(r*cos(theta))+r*sin(theta);
```

$delr := \frac{1}{10}$

$deltheta := \frac{1}{20} \pi$

$G := (r, \theta) \rightarrow \sin(r \cos(\theta)) + r \sin(\theta)$

```
>
```

```
> Sum(Sum('G((i-1)*delr+delr/2,(j-1)*deltheta+deltheta/2)*delr*deltheta*((i-1)*delr+delr/2)',
```

```
'i'=1..N), 'j'=1..N)=evalf(sum(sum('G((i-1)*delr+delr/2, (j-1)*deltheta+deltheta/2)*delr*deltheta*((i-1)*delr+delr/2)', 'i'=1..N), 'j'=1..N));
```

$$\sum_{j=1}^{10} \left(\sum_{i=1}^{10} G \left((i-1) \text{delr} + \frac{1}{2} \text{delr}, (j-1) \text{deltheta} + \frac{1}{2} \text{deltheta} \right) \text{delr} \text{deltheta} \left((i-1) \text{delr} + \frac{1}{2} \text{delr} \right) \right) = .5617990262$$

```
>
```

```
> Int( Int(r*G(r,theta), theta=0 .. Pi/2), r=0..1) =int( int(r*G(r,theta), theta=0 ..Pi/2), r=0..1) ;
```

$$\int_0^1 \int_0^{1/2\pi} r \sin(r \cos(\theta) + r \sin(\theta)) d\theta dr = \int_0^1 \int_0^{1/2\pi} r \sin(r \cos(\theta) + r \sin(\theta)) d\theta dr$$

```
> Int( Int(r*G(r,theta), r=0 .. 1), theta=0 .. Pi/2) =int( int(r*G(r,theta), r=0 ..1), theta=0 ..Pi/2) ;
```

$$\int_0^{1/2\pi} \int_0^1 r \sin(r \cos(\theta) + r \sin(\theta)) dr d\theta = \int_0^{1/2\pi} (\sin(\cos(\theta)) \cos(\sin(\theta)) + \cos(\cos(\theta)) \sin(\sin(\theta)) - \cos(\theta) \cos(\cos(\theta)) \cos(\sin(\theta)) + \cos(\theta) \sin(\cos(\theta)) \sin(\sin(\theta)) - \sin(\theta) \cos(\cos(\theta)) \cos(\sin(\theta)) + \sin(\theta) \sin(\cos(\theta)) \sin(\sin(\theta))) / (\cos(\theta) + \sin(\theta))^2 d\theta$$

```
> # silo problem use sym
```

```
> N:=10;
```

$$N := 10$$

```
> delr:=5/N;deltheta:=Pi/2/N;
```

```
> G:= (r,theta)-> 40 +sqrt(25-r^2);
```

$$\text{delr} := \frac{1}{2}$$

$$\text{deltheta} := \frac{1}{20} \pi$$

$$G := (r, \theta) \rightarrow 40 + \sqrt{25 - r^2}$$

```
>
```

```
> Sum(Sum('G((i-1)*delr+delr/2, (j-1)*deltheta+deltheta/2)*delr*deltheta*((i-1)*delr+delr/2)', 'i'=1..N), 'j'=1..N)=evalf(sum(sum('G((i-1)*delr+delr/2, (j-1)*deltheta+deltheta/2)*delr*deltheta*((i-1)*delr+delr/2)', 'i'=1..N), 'j'=1..N));
```

$$\sum_{j=1}^{10} \left(\sum_{i=1}^{10} G \left((i-1) \text{delr} + \frac{1}{2} \text{delr}, (j-1) \text{deltheta} + \frac{1}{2} \text{deltheta} \right) \text{delr} \text{deltheta} \left((i-1) \text{delr} + \frac{1}{2} \text{delr} \right) \right) = 851.4463383$$

```
>
```

```
> Int( Int(r*G(r,theta), theta=0 .. Pi/2), r=0..5) =int( int(r*G(r,theta), theta=0 ..Pi/2), r=0..5);evalf(%) ;
```

$$\int_0^5 \int_0^{1/2\pi} r (40 + \sqrt{25 - r^2}) d\theta dr = \frac{1625}{6} \pi$$

$$850.8480103 = 850.8480104$$

```
> Int( Int(r*G(r,theta), r=0 .. 5), theta=0 .. Pi/2) =int( int(r*G(r,theta), r=0 ..5), theta=0 ..Pi/2) ;evalf(%) ;
```

$$\int_0^{1/2\pi} \int_0^5 r (40 + \sqrt{25 - r^2}) dr d\theta = \frac{1625}{6} \pi$$

$$850.8480103 = 850.8480104$$

```
>
```