```
> restart:with(plots):
```

```
Warning, the name changecoords has been redefined
```

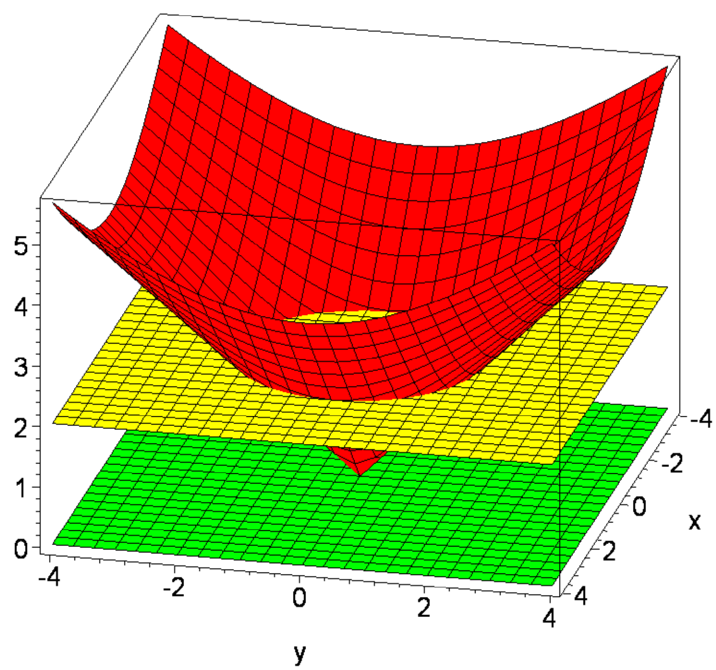
Section I ex 1

```
> g := (x, y) -> sqrt(x^2+y^2);
  h:= (x,y) -> 2; p:= (x,y) -> 0;  plot1:=plot3d(h(x,y), x = -4..4,y
  = -4 .. 4, color=yellow):plot2:= plot3d(p(x,y), x = -4..4, y = -4
  .. 4, color=green):
  plot3:=plot3d(g(x,y), x = -4..4, y = -4 .. 4, color=red):
  display3d({plot1,plot2,plot3}, axes=boxed);
```

$$g := (x, y) \to \sqrt{x^2 + y^2}$$

$$h := 2$$

$$p := 0$$



Find volume between g(x,y) and h(x,y)

```
> 4* Int(Int( 2- sqrt(x^2+y^2), y=0..sqrt(2-x^2)),x=0..sqrt(2))=

  4* int(int( 2- sqrt(x^2+y^2), y=0..sqrt(2-x^2)),x=0..sqrt(2));
  4* Int(Int(Int( 1,z =sqrt(x^2+y^2).. 2),
  y=0..sqrt(2-x^2)),x=0..sqrt(2))=

  4* int(int( int(1, z = sqrt(x^2+y^2) .. 2),
```

```
y=0..sqrt(2-x^2)),x=0..sqrt(2));
```

$$4\int_0^{\sqrt{2}}\int_0^{\sqrt{2-x^2}} 2-\sqrt{x^2+y^2}\,dy\,dx =$$

$$4\int_0^{\sqrt{2}} -\frac{1}{2}x^2\ln(\sqrt{2-x^2}+\sqrt{2})+2\sqrt{2-x^2}-\frac{1}{2}\sqrt{2-x^2}\sqrt{2}+\frac{1}{4}x^2\ln(x^2)\,dx$$

$$4\int_0^{\sqrt{2}}\int_0^{\sqrt{2-x^2}}\int_{\sqrt{x^2+y^2}}^{2} 1\,dz\,dy\,dx =$$

$$4\int_0^{\sqrt{2}} -\frac{1}{2}x^2\ln(\sqrt{2-x^2}+\sqrt{2})+2\sqrt{2-x^2}-\frac{1}{2}\sqrt{2-x^2}\sqrt{2}+\frac{1}{4}x^2\ln(x^2)\,dx$$

TROUBLE! so try polars/cylindricals

```
> Int(Int((2 - r)*r, r= 0..2),theta =0..2*Pi)=
  int(int((2 - r)*r, r= 0..2),theta =0..2*Pi);
  Int(Int( Int ( r,z = r .. 2), r= 0..2),theta =0..2*Pi)=
  int(int( int ( r,z = r .. 2), r= 0..2),theta =0..2*Pi);
>
```

$$\int_0^{2\pi}\int_0^2 (2-r)\,r\,dr\,d\theta = \frac{8}{3}\pi$$

$$\int_0^{2\pi}\int_0^2\int_r^2 r\,dz\,dr\,d\theta = \frac{8}{3}\pi$$

ex 2 vol bounded by cone and sphere of radius 2 great place for sphereical coordinates

```
> Int(Int(Int(rho^2*sin(phi), rho=0..2),
  phi=0..Pi/4),theta=0..Pi*2)= int(int(int(rho^2*sin(phi),
  rho=0..2), phi=0..Pi/4),theta=0..Pi*2);
```

$$\int_0^{2\pi}\int_0^{1/4\,\pi}\int_0^2 \rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta = -\frac{8}{3}\sqrt{2}\,\pi+\frac{16}{3}\pi$$

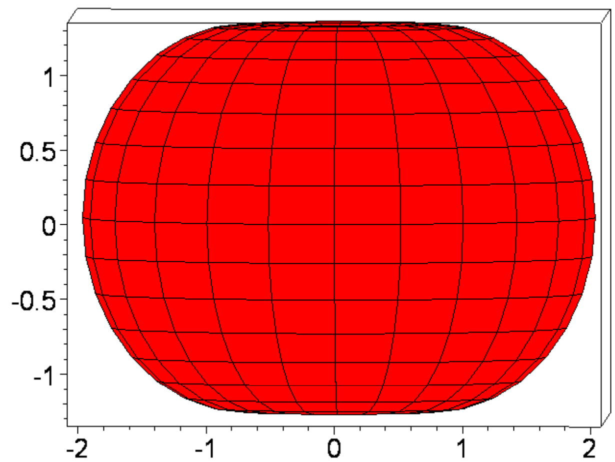
SEction II ex1

```
> Int(Int(Int(rho^2*sin(phi), rho=1..1 + sin(phi)),
  phi=0..Pi/2),theta=0..Pi*2)= int(int(int(rho^2*sin(phi), rho=1..1
  + sin(phi)), phi=0..Pi/2),theta=0..Pi*2);
```

$$\int_0^{2\pi}\int_0^{1/2\,\pi}\int_1^{1+\sin(\phi)} \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta = \frac{5}{8}\pi^2 + \frac{4}{3}\pi$$

but what is it

```
> plot3d(1+sin(phi), theta=0..2*Pi, phi=0..Pi,
  coords=spherical,color = red, axes=boxed);
```



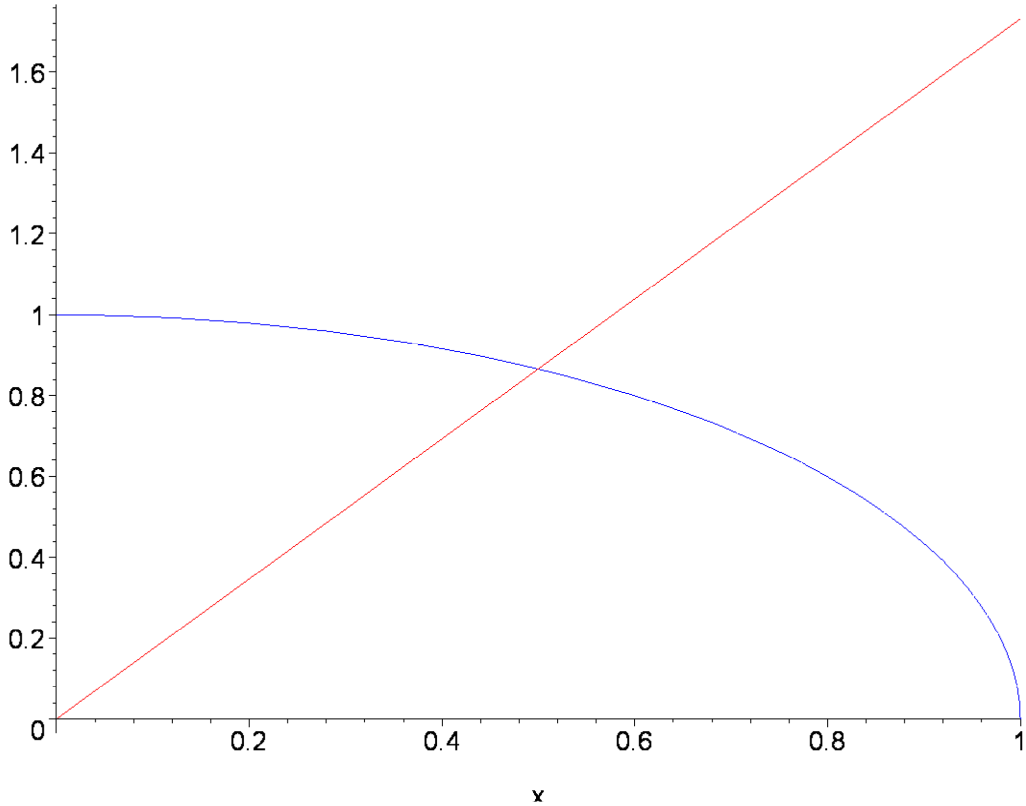
$$\int_0^{2\pi}\int_0^{\pi}\int_0^{2} \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta = \frac{32}{3}\pi$$

ex 2

Find D

$$r0 := 1 - \sin(\theta)$$

```
> p1 :=plot(sqrt(3)*x, x = 0 ..1 ,color = red):p2 :=implicitplot(x^2
  + y^2 = 1,x=-1..1,y=0..1,color = blue): p3 :=plot(0, x = 0 ..1
  ,color = green):
> display(p1,p2,p3);
```




```
> Int(Int(r, r=0..1 ), theta=0..Pi/3)= int(int(r, r=0..1 ),
  theta=0..Pi/3);
```

$$\int_0^{1/3\,\pi}\int_0^1 r\,dr\,d\theta=\frac{1}{6}\pi$$

ex 3

```
> Int(Int(Int(1, z=0..sqrt(1-x^2-y^2)),
  x=y..sqrt(1-y^2)),y=0..sqrt(2)/2)=int(int(int(1,
  z=0..sqrt(1-x^2-y^2)), x=y..sqrt(1-y^2)),y=0..sqrt(2)/2);
```

$$\int_0^{1/2\sqrt{2}}\int_y^{\sqrt{1-y^2}}\int_0^{\sqrt{1-x^2-y^2}} 1\,dz\,dx\,dy=\frac{1}{12}\pi$$

so maple takes a while , let's do it in sphereical coorinates

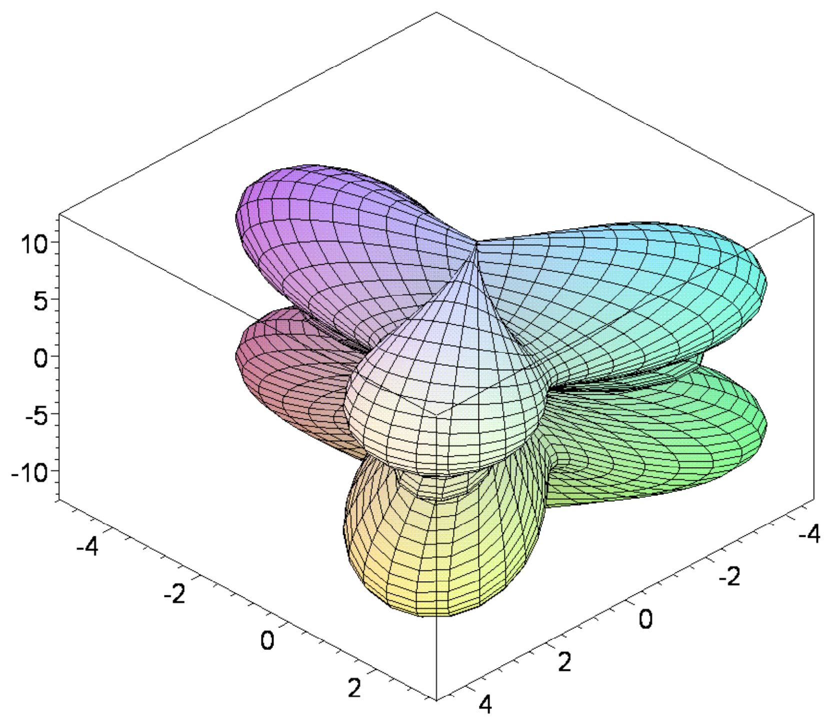
```
> Int(Int(Int(rho^2*sin(phi), rho=0..1 ),
  phi=0..Pi/2),theta=0..Pi/4)= int(int(int(rho^2*sin(phi), rho=0..1
  ), phi=0..Pi/2),theta=0..Pi/4);
```

$$\int_0^{1/4\pi}\int_0^{1/2\pi}\int_0^1 \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta = \frac{1}{12}\pi$$

ex4

```
> Int(Int(Int(1, z=0..sqrt(1-x^2-y^2)),
  y=0..sqrt(1-x^2)),x=0..1)=int(int(int(1, z=0..sqrt(1-x^2-y^2)),
  y=0..sqrt(1-x^2)),x=0..1);
```

$$\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} 1\, dz\, dy\, dx = \frac{1}{6}\pi$$



ex5

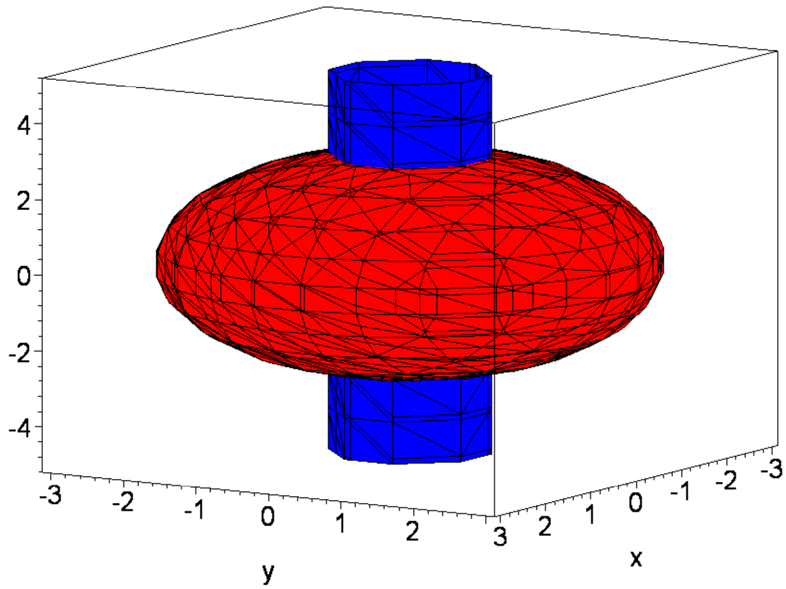
```
> Int(Int(Int(rho^2*sin(phi), rho=0 ..2),  phi=0..Pi/2),theta
  =0..Pi)=
  int(int(int(rho^2*sin(phi), rho=0 ..2),  phi=0..Pi/2),theta
  =0..Pi);
```

$$\int_0^{\pi}\int_0^{1/2\pi}\int_0^2 \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta = \frac{8}{3}\pi$$

ex6

```
> plot1:=implicitplot3d( x^2 + y^2 + z^2=9,x=-3..3,y=-3..3,
  z=-3..3,color=red): plot2 :=
```

```
implicitplot3d( x^2 + y^2 =1,x=-3..3,y=-3..3,
z=-5..5,color=blue):display({plot1,plot2},axes=boxed);
```



let's try cylindrical coordinates top issphere and use sym 8 x

```
>
> 8*Int(Int(Int(r, z=0 .. sqrt(9-r^2)), r=0 .. 1),
  theta=0..Pi/2)=8*int(int(int(r, z=0 .. sqrt(9-r^2)), r=0 .. 1),
  theta=0..Pi/2);
```

$$8 \int_{0}^{1/2\,\pi} \int_{0}^{1} \int_{0}^{\sqrt{9-r^2}} r \, dz \, dr \, d\theta = -\frac{64}{3}\sqrt{2}\,\pi + 36\,\pi$$

ex 7

```
> Int(Int(Int(rho^2*sin(phi), rho=0..2),
  phi=0..Pi/6),theta=0..2*Pi)=int(int(int(rho^2*sin(phi), rho=0..2),
  phi=0..Pi/6),theta=0..2*Pi);
>
>
```

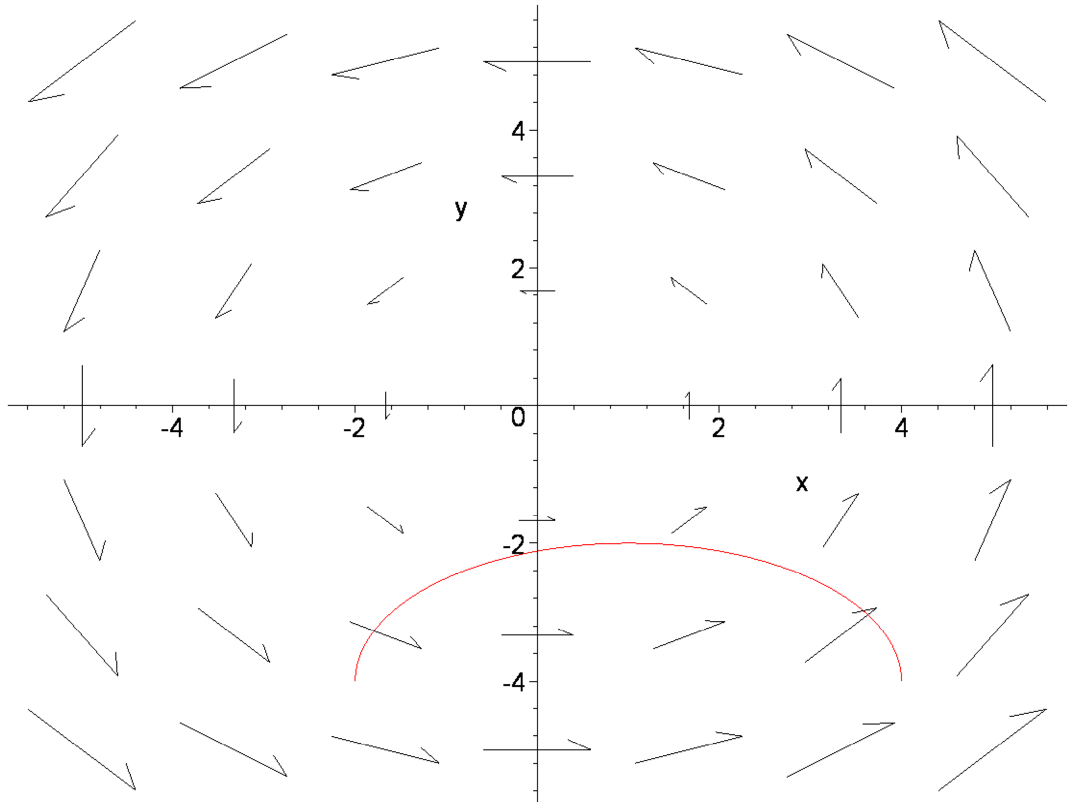
$$\int_{0}^{2\,\pi} \int_{0}^{1/6\,\pi} \int_{0}^{2} \rho^2 \sin(\phi) \, d\rho \, d\phi \, d\theta = -\frac{8}{3}\sqrt{3}\,\pi + \frac{16}{3}\,\pi$$

>

$$vfield := [-y, x]$$

$$path := [1 + 3\cos(t), -4 + 2\sin(t)]$$

$$trange := t = 0 .. \pi$$



the vector field is , $[-y, x]$

the path is , $[1 + 3\cos(t), -4 + 2\sin(t)]$, *with* , $t = 0 .. \pi$

$$\int_0^{\pi} [-y, x] \left(\frac{\partial}{\partial t} [1 + 3\cos(t), -4 + 2\sin(t)] \right) dt$$

the funtion on the path is , $[4 - 2\sin(t), 1 + 3\cos(t)]$

$$\int_0^{\pi} [4 - 2\sin(t), 1 + 3\cos(t)] \left(\frac{\partial}{\partial t} [1 + 3\cos(t), -4 + 2\sin(t)] \right) dt$$

the derivative of the path is , $[-3\sin(t), 2\cos(t)]$

$$\int_0^{\pi} [4 - 2\sin(t), 1 + 3\cos(t)] [-3\sin(t), 2\cos(t)] \, dt$$

The integrand is , $-12\sin(t) + 2\cos(t) + 6$

$$\int_0^{\pi} -12\sin(t) + 2\cos(t) + 6\, dt$$

the integral is , $-24 + 6\pi$

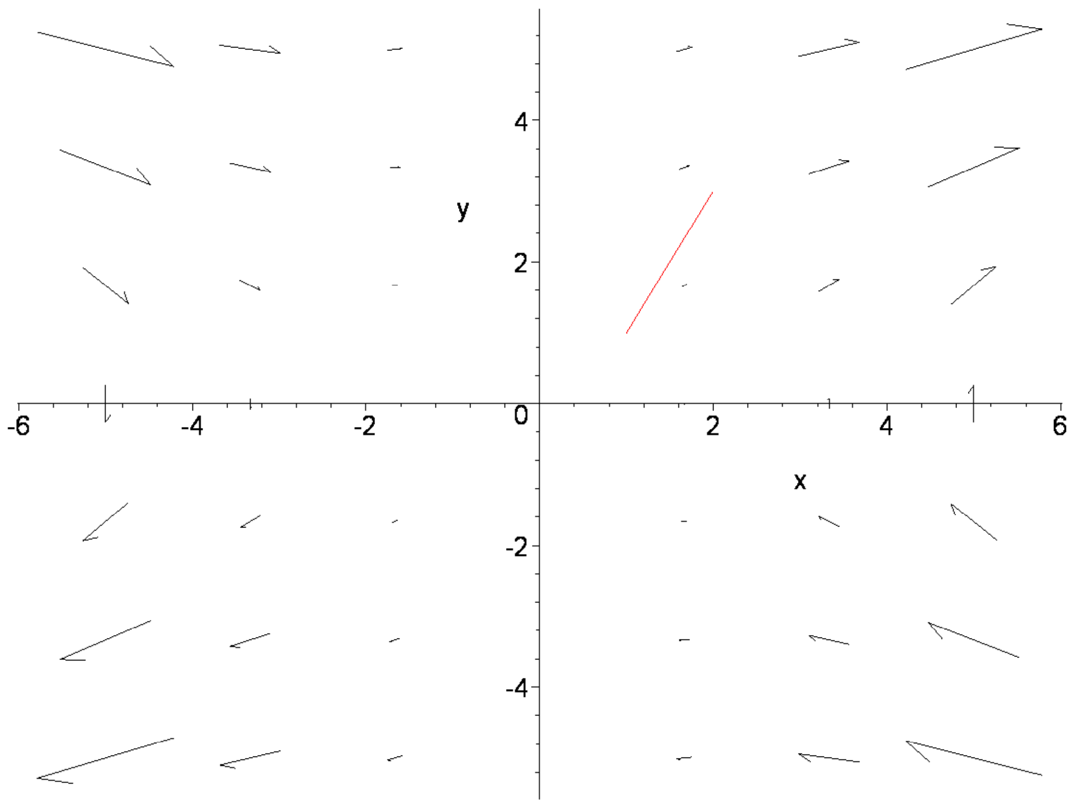
-5.15044408

```
> vfield := [3*x^2*y,x^3+2*y];
  path := [t+1, 2*t+1];
  trange := t=0..1;
  pathplot(vfield, path, trange, x=-5..5, y=-5..5);
  lineintegral(vfield, path, trange);
```

$$vfield := [3\,x^2\,y, x^3 + 2\,y]$$

$$path := [t+1, 2\,t+1]$$

$$trange := t = 0 .. 1$$



the vector field is , $[3\,x^2\,y, x^3 + 2\,y]$

the path is , $[t+1, 2\,t+1]$, *with* , $t = 0 .. 1$

$$\int_0^1 [3\,x^2\,y, x^3 + 2\,y]\left(\frac{\partial}{\partial t}[t+1, 2\,t+1]\right) dt$$

the funtion on the path is , $[3\,(t+1)^2\,(2\,t+1), (t+1)^3 + 4\,t + 2]$

$$\int_0^1 [3\,(t+1)^2\,(2\,t+1), (t+1)^3+4\,t+2]\left(\frac{\partial}{\partial t}[t+1, 2\,t+1]\right) dt$$

the derivative of the path is , [1, 2]

$$\int_0^1 [3\,(t+1)^2\,(2\,t+1), (t+1)^3+4\,t+2]\,[1, 2]\, dt$$

The integrand is , $8\,t^3+21\,t^2+26\,t+9$

$$\int_0^1 8\,t^3+21\,t^2+26\,t+9\, dt$$

the integral is , 31

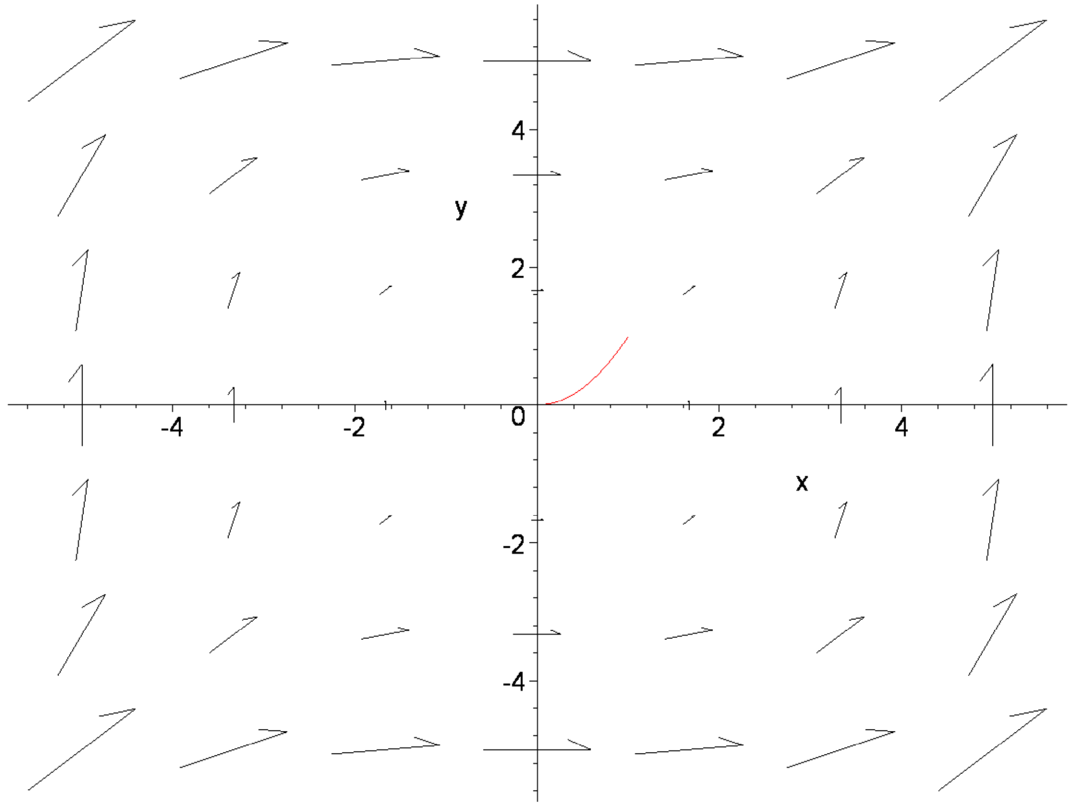
31.

```
> vfield := [y^2,x^2];
  path := [t, t^2];
  trange := t=0..1;
  pathplot(vfield, path, trange, x=-5..5, y=-5..5);
  lineintegral(vfield, path, trange);
```

$$vfield := [y^2, x^2]$$

$$path := [t, t^2]$$

$$trange := t = 0\,..\,1$$



the vector field is , $[y^2, x^2]$

the path is , $[t, t^2]$, *with* , $t = 0 .. 1$

$$\int_0^1 [y^2, x^2]\left(\frac{\partial}{\partial t}[t, t^2]\right) dt$$

the funtion on the path is , $[t^4, t^2]$

$$\int_0^1 [t^4, t^2]\left(\frac{\partial}{\partial t}[t, t^2]\right) dt$$

the derivative of the path is , $[1, 2\,t]$

$$\int_0^1 [t^4, t^2]\,[1, 2\,t]\, dt$$

The integrand is , $t^4 + 2\,t^3$

$$\int_0^1 t^4 + 2\,t^3\, dt$$

the integral is , $\frac{7}{10}$

.7000000000

>