Name ______________________________ April 7,2004

Math. 2686h Exam II

I. Answer the following:

1.) This exercise is to illustrate an important point concerning continuity and having a partial derivative. For extra credit please describe your reactions to this problem.

a.) Find $Lim_{(x,y)->(0,0)}$ for $f(x,y) = \begin{cases} \dfrac{x*y}{x^2+y^2} \; for(x,y) \neq (0,0) \\ 0, for \; (x,y) = (0,0) \end{cases}$

b.) Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ for $(x,y) \neq (0,0)$.

c.) Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ for $(x,y) = (0,0)$. Hint: direct definition is needed .

d.) Comment!

2.) For the intersection of the surface $z = x^2 + y^2$ and the plane $z = 1$ find the rate of change of the tangent line to the curve of at the point $(a, b, 1)$.

3.) Find the all critical points for the following function and label them as local maximum and minimum values and saddle points accordingly where
$f(x,y) = 4xy - x^4 - y^4 + 1$

4.) Find the equation of the tangent plane at the point $P = (-3, 1, -3)$ to the ellipsoid $x^2/9 + y^2 + z^2/9 = 3$ (2 ways)

a.) First consider $z = f(x,y)$ and do it the way we have done all these problems. Be sure you select the correct sign on the square root.

b.) Second consider $k = f(x,y,z)$ and use the formula $0 = \dfrac{\partial f}{\partial x}(x - x_o) + \dfrac{\partial f}{\partial y}(y - y_o) + \dfrac{\partial f}{\partial z}(z - z_o)$ where all the partials are evaluated at P_o. Hopefully you get the same answer.

5.) For $u = x^4 * y + y^2 * z^3$ and $x = r * s * e^t$, $y = r * s^2 * e^{-t}$ and $z = r * s * \sin(t)$ find $\dfrac{\partial u}{\partial s}$.

6.) Temperature is given by $T(x, y, z) = \dfrac{80}{1 + x^2 + 2y^2 + 3z^2}$ where $\bigtriangledown T = \dfrac{\partial T}{\partial x}i + \dfrac{\partial T}{\partial y}j + \dfrac{\partial T}{\partial z}k$

a.) In what direction does T increase most rapidly?

b.) What is that increase?

7.) Use Lagrange multipliers to find the dimensions of the maximum rectangular box with "no lid" which is made from $12ft^2$ of cardboard.

8.) Use Lagrange multipliers to find the dimensions of the optimally constructed silo with a hemi-sphereical top to hold 8000 cubic feet

9.) For $z = x^2 + 3xy - y^2$ find the tangent plane at (2,3).

a.) Use the this to approximate the value at $(2.05, 2.96)$

b.) What is the error?