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[ > restart:
[ > with (student):with(plots):
  with(linalg):
Warning, the name changecoords has been redefined
Warning, the protected names norm and trace have been redefined and unprotected
[ ex 1
let's try y=mx
[ > f:= (x,y)-> (x*y)/(x^2+y^2);limit(f(x,y), {x=0,y=0});


$$f := (x, y) \rightarrow \frac{xy}{x^2 + y^2}$$


$$\lim\left(\frac{xy}{x^2 + y^2}, \{x=0, y=0\}\right)$$

[ > limit((m*x*x)/(x^2+m^2*x^2), {x=0});


$$\frac{m}{1 + m^2}$$

[ > Diff(f(x,y), x) = diff(f(x,y), x);


$$\frac{\partial}{\partial x} \frac{xy}{x^2 + y^2} = \frac{y}{x^2 + y^2} - \frac{2x^2 y}{(x^2 + y^2)^2}$$

[ > Diff(f(x,y), y) = diff(f(x,y), y);


$$\frac{\partial}{\partial y} \frac{xy}{x^2 + y^2} = \frac{x}{x^2 + y^2} - \frac{2xy^2}{(x^2 + y^2)^2}$$

[ > limit(f(0+h,0)/h, {h=0});

0
[ > limit(f(0,0+h)/h, {h=0});

0
[ >
[ ex2
[ > v:=array(1..3, [0,0,0]);F:=(x,y,z)-> x*x+y^2-z;

v := [0, 0, 0]

$$F := (x, y, z) \rightarrow x^2 + y^2 - z$$

[ > grad(F(x,y,z), vector([x,y,z]));

[2 x, 2 y, -1]
[ > v:=subs(x=a,y=b,z=1,grad(F(x,y,z), vector([x,y,z])));

v := [2 a, 2 b, -1]
[ > u:=crossprod(v, [0,0,1]);g1:=array(1..3, [0,0,0]);gp1:=array(1..3, [0,0,0]);

u := [2 b, -2 a, 0]

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> tanline := t -> [t*u[1]+a,t*u[2]+b,t*u[3]+1];evalf(tanline(t));
      
$$\text{tanline} := t \rightarrow [t u_1 + a, t u_2 + b, t u_3 + 1]$$

      
$$[2. t b + a, -2. t a + b, 1.]$$

> g:=
theta->[a*cos(theta)*sqrt(2),b*sin(theta)*sqrt(2),1];subs(theta=Pi
/4,g(theta));g1:=evalf(%);
      
$$g := \theta \rightarrow [a \cos(\theta) \sqrt{2}, b \sin(\theta) \sqrt{2}, 1]$$

      
$$\left[ a \cos\left(\frac{1}{4} \pi\right) \sqrt{2}, b \sin\left(\frac{1}{4} \pi\right) \sqrt{2}, 1 \right]$$

      
$$g1 := [a, b, 1.]$$

> gp:=diff(g(theta),theta);subs(theta=Pi/4,gp(theta));gp1:=evalf(%);
      
$$gp := [-a \sin(\theta) \sqrt{2}, b \cos(\theta) \sqrt{2}, 0]$$

      
$$\left[ -a \left(\frac{1}{4} \pi\right) \sin\left(\frac{1}{4} \pi\right) \sqrt{2}, b \left(\frac{1}{4} \pi\right) \cos\left(\frac{1}{4} \pi\right) \sqrt{2}, 0 \right]$$

      
$$gp1 := \left[ -1. a \left(\frac{1}{4} \pi\right), b \left(\frac{1}{4} \pi\right), 0. \right]$$

[ tangent line
> tanline := t -> [t*gp1[1]+g1[1],t*gp1[2]+g1[2],t*gp1[3]+g1[3]];
      
$$\text{tanline} := t \rightarrow [t gp1_1 + g1_1, t gp1_2 + g1_2, t gp1_3 + g1_3]$$

> evalf(tanline(t));
      
$$\left[ -1. t a \left(\frac{1}{4} \pi\right) + a, t b \left(\frac{1}{4} \pi\right) + b, 1. \right]$$

[ ex3
> f:=(x,y)-> 4*x*y-x^4-y^4 +1 ;
      
$$f := (x, y) \rightarrow 4 y x - x^4 - y^4 + 1$$

> Diff(f(x,y),x) = diff(f(x,y),x);Diff(f(x,y),y) = diff(f(x,y),y);
      
$$\frac{\partial}{\partial x} (4 y x - x^4 - y^4 + 1) = 4 y - 4 x^3$$

      
$$\frac{\partial}{\partial y} (4 y x - x^4 - y^4 + 1) = 4 x - 4 y^3$$

> solve({diff(f(x,y),x)=0,diff(f(x,y),y)=0}, {x,y});
      
$$\{y = 0, x = 0\}, \{x = -\text{RootOf}(\_Z^2 + 1, \text{label} = \_L4), y = \text{RootOf}(\_Z^2 + 1, \text{label} = \_L4)\},$$

      
$$\{x = 1, y = 1\}, \{x = -1, y = -1\}, \{$$

      
$$x = \text{RootOf}(-\text{RootOf}(\_Z^2 + 1, \text{label} = \_L3) + \_Z^2, \text{label} = \_L5) \text{RootOf}(\_Z^2 + 1, \text{label} = \_L3),$$

      
$$y = \text{RootOf}(-\text{RootOf}(\_Z^2 + 1, \text{label} = \_L3) + \_Z^2, \text{label} = \_L5)\}$$

> disc:=(x,y)->
diff(f(x,y),x,x)*diff(f(x,y),y,y)-diff(f(x,y),x,y)*diff(f(x,y),x,y
);

```

$$\text{disc} := (x, y) \rightarrow \text{diff}(f(x, y), x, x) \text{diff}(f(x, y), y, y) - \text{diff}(f(x, y), x, y)^2$$

> subs(x=0, y=0, disc(x,y));

-16

> subs(x=1, y=1, disc(x,y));subs(x=1, y=1, diff(f(x,y),x,x));

128

-12

> subs(x=-1, y=-1, disc(x,y));subs(x=-1, y=-1, diff(f(x,y),x,x));

128

-12

ex #4

> restart:with (student):

> f:=(x,y)-> -sqrt(27 -x^2-9*y^2);

$$f := (x, y) \rightarrow -\sqrt{27 - x^2 - 9y^2}$$

> z:= -3 + (x+3)*

subs(x=-3,y=1,diff(f(x,y),x))+(y-1)*subs(x=-3,y=1,diff(f(x,y),y));

$$z := -3 - \frac{1}{3}(x+3)\sqrt{9} + (y-1)\sqrt{9}$$

> z:=simplify(%);

$$z := -9 - x + 3y$$

> restart:with (student):f:=(x,y,z)-> x^2/9+y^2+z^2/9-3;

$$f := (x, y, z) \rightarrow \frac{1}{9}x^2 + y^2 + \frac{1}{9}z^2 - 3$$

> fx:=

subs(x1=-3,y1=1,z1=-3,2*x1/9);fy:=subs(x1=-3,y1=1,z1=-3,2*y1);fz:=
subs(x1=-3,y1=1,z1=-3,2*z1/9);

$$fx := \frac{-2}{3}$$

$$fy := 2$$

$$fz := \frac{-2}{3}$$

> fx*(x+3)+fy*(y-1)+fz*(z+3)=0;

$$-\frac{2}{3}x - 6 + 2y - \frac{2}{3}z = 0$$

> simplify(%);

$$-\frac{2}{3}x - 6 + 2y - \frac{2}{3}z = 0$$

ex 5

> restart:with (student):U:= (x,y,z)->x^4*y+y^2*z^3;

$$U := (x, y, z) \rightarrow x^4 y + y^2 z^3$$

```
> X := (r, s, t) -> r*s*exp(t); Y := (r, s, t) -> r*s^2*exp(-t); Z := (r, s, t) -> r*s*sin(t);
```

$$X := (r, s, t) \rightarrow r s e^t$$

$$Y := (r, s, t) \rightarrow r s^2 e^{-t}$$

$$Z := (r, s, t) \rightarrow r s \sin(t)$$

```
> Diff(U(x,y,z), x)*Diff(X(r,s,t), s)+Diff(U(x,y,z), y)*Diff(Y(r,s,t), s)+Diff(U(x,y,z), z)*Diff(Z(r,s,t), t)=diff(U(x,y,z), x)*diff(X(r,s,t), s)+diff(U(x,y,z), y)*diff(Y(r,s,t), s)+diff(U(x,y,z), z)*diff(Z(r,s,t), t);diff(U(x,y,z), x)*diff(X(r,s,t), s)+diff(U(x,y,z), y)*diff(Y(r,s,t), s)+diff(U(x,y,z), z)*diff(Z(r,s,t), t);
```

$$\left(\frac{\partial}{\partial x}(x^4 y + y^2 z^3)\right)\left(\frac{\partial}{\partial s} r s e^t\right) + \left(\frac{\partial}{\partial y}(x^4 y + y^2 z^3)\right)\left(\frac{\partial}{\partial s} r s^2 e^{-t}\right) + \left(\frac{\partial}{\partial z}(x^4 y + y^2 z^3)\right)\left(\frac{\partial}{\partial t} r s \sin(t)\right) = 4 x^3 y r e^t + 2 (x^4 + 2 y z^3) r s e^{-t} + 3 y^2 z^2 r s \cos(t)$$

$$4 x^3 y r e^t + 2 (x^4 + 2 y z^3) r s e^{-t} + 3 y^2 z^2 r s \cos(t)$$

```
> subs(x=r*s*exp(t), y=r*s^2*exp(-t), z=r*s*sin(t), %);
```

$$4 r^5 s^5 (e^t)^4 e^{-t} + 2 (r^4 s^4 (e^t)^4 + 2 r^4 s^5 e^{-t} \sin(t)^3) r s e^{-t} + 3 r^5 s^7 (e^{-t})^2 \sin(t)^2 \cos(t)$$

```
> simplify(%);
```

$$-r^5 s^5 e^{-t} (-6 e^{4t} - 4 s e^{-t} \sin(t) + 4 s e^{-t} \sin(t) \cos(t)^2 - 3 s^2 e^{-t} \cos(t) + 3 s^2 e^{-t} \cos(t)^3)$$

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ex 6
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> restart:with(student):with(linalg):T := (x,y,z) -> 80/(1 + x^2 + 2*y^2 + 3*z^2);
```

Warning, the protected names norm and trace have been redefined and unprotected

$$T := (x, y, z) \rightarrow 80 \frac{1}{1 + x^2 + 2 y^2 + 3 z^2}$$

```
> v := subs(x=1, y=1, z=1, grad(T(x,y,z), vector([x,y,z])));
```

$$v := \left[\frac{-160}{49}, \frac{-320}{49}, \frac{-480}{49} \right]$$

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> norm(v, 2);
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$$\frac{160}{49} \sqrt{14}$$

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ex 7
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> restart:with(student):with(linalg):S := (x,y,z) -> x*y + 2*x*z + 2*y*z - 12;
```

Warning, the protected names norm and trace have been redefined and unprotected

$$S := (x, y, z) \rightarrow y x + 2 x z + 2 y z - 12$$

```

> V:=(x,y,z)-> x*y*z;
                                V := (x, y, z) -> x y z
> L:=(x,y,z,lambda)-> V(x,y,z)+lambda*S(x,y,z);
                                L := (x, y, z, λ) -> V(x, y, z) + λ S(x, y, z)
> diff(L(x,y,z,lambda),x)=0;
                                y z + λ (y + 2 z) = 0
> diff(L(x,y,z,lambda),y)=0;
                                x z + λ (x + 2 z) = 0
> diff(L(x,y,z,lambda),z)=0;
                                y x + λ (2 x + 2 y) = 0
> diff(L(x,y,z,lambda),lambda)=0;
                                y x + 2 x z + 2 y z - 12 = 0
> solve({diff(L(x,y,z,lambda),x)=0,
diff(L(x,y,z,lambda),y)=0,diff(L(x,y,z,lambda),z)=0,diff(L(x,y,z,lambda),lambda)=0},{x,y,z,lambda});
                                {z = 1, λ = -1/2, y = 2, x = 2}, {λ = 1/2, y = -2, x = -2, z = -1}

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ex 8

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> restart:with(student):with(linalg):S:=(r,h)->2*Pi*r^2+2*Pi*h*r;
Warning, the protected names norm and trace have been redefined and unprotected
                                S := (r, h) -> 2 π r2 + 2 π h r
> V:=(r,h)-> 2/3*Pi*r^3+Pi*r^2*h-8000;
                                V := (r, h) -> 2/3 π r3 + π r2 h - 8000
> L:=(r,h,lambda)-> S(r,h)+lambda*V(r,h);
                                L := (r, h, λ) -> S(r, h) + λ V(r, h)
> diff(L(r,h,lambda),r)=0;
                                4 π r + 2 π h + λ (2 π r2 + 2 π h r) = 0
> diff(L(r,h,lambda),h)=0;
                                2 π r + λ π r2 = 0
                                y x + λ (2 x + 2 y) = 0
> diff(L(r,h,lambda),lambda)=0;
                                2/3 π r3 + π r2 h - 8000 = 0
> solve({diff(L(r,h,lambda),r)=0,
diff(L(r,h,lambda),h)=0,diff(L(r,h,lambda),lambda)=0},{r,h,lambda});
                                {h = 0, λ = -1/60 π RootOf(π _Z3 - 12, label = _L2)2, r = 10 RootOf(π _Z3 - 12, label = _L2)}

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[ex 9

> **restart:with (student):with(linalg):**

Warning, the protected names norm and trace have been redefined and unprotected

>

> **f:=(x,y)->x^2 +**

3*x*y-y^2;fx:=subs(x=2,y=3,diff(f(x,y),x));fy:=subs(x=2,y=3,diff(f(x,y),y));

$$f := (x, y) \rightarrow x^2 + 3yx - y^2$$

$$fx := 13$$

$$fy := 0$$

> **z:= +f(2,3) + fx*(x-2) + (y-1)*fy;**

$$z := -13 + 13x$$

> **z:=subs(x=2.05,y=2.96,evalf(%));**

$$z := 13.65$$

> **evalf(%) - f(2.05, 2.96);**

$$.0051$$

[>