> restart; with(linalg);
Warning, the protected names norm and trace have been redefined and unprotected

Ex1
> U := transpose(matrix([[2, 3, 2], [7, 10, 6], [6, 10, 7]]));

U :=

\[
\begin{bmatrix}
2 & 7 & 6 \\
3 & 10 & 10 \\
2 & 6 & 7
\end{bmatrix}
\]

a.) Prove that the columns of U form a basis for \( \mathbb{R}^3 \)

b.) Let \( \mathbf{w} \) be a vector in \( \mathbb{R}^3 \) whose coordinates with respect to the U basis is \([3, 5, 6]\)

b1.) Find the coordinates of \( \mathbf{w} \) with respect to the standard basis.

c.) Let \( \mathbf{w} \) be a vector in \( \mathbb{R}^3 \) whose coordinates with respect to the standard basis is \([3, 5, 6]\).

Find the coordinates of \( \mathbf{w} \) with respect to the U basis.
> wU := vector([[3, 5, 6]]);

wU := [3, 5, 6]

Ex2
Consider the space \( P_2 = \{ ax^2 + bx + c \mid a, b, c \in \mathbb{R} \}, \) polynomials of degree 2.

Let us consider two basis for \( P_2 \)

Natural basis \( N = \{ x^2, x, 1 \} \) and the \( H = \{ x^2 - 3x + 2, x^2 - 2x, x^2 - x \} \)

a.) Prove that \( N \) and \( H \) form a basis for \( P_2 \)

b.) Find the coordinates of elements of \( H \) with respect to the \( N \) basis

c.) Find the coordinates of elements of \( N \) with respect to the \( H \) basis

d.) Let \( \mathbf{w} \) be a vector in \( P_2 \) whose coordinates with respect to the \( H \) basis is \([12, -11, 5]\)

d1.) Find the coordinates of \( \mathbf{w} \) with respect to the \( N \) basis.

e.) Let \( \mathbf{w} \) be a vector in \( P_2 \) whose coordinates with respect to the \( N \) basis is \([12, -11, 5]\)

e1.) Find the coordinates of \( \mathbf{w} \) with respect to the \( H \) basis.

Ex3
> A := (matrix([[17, 12, 18], [-16, -9, -24], [-5, -4, -4]])); C :=

transpose( matrix([[10, -8, -3], [-3, 3, 1], [-3, 2, 1]]));

A :=

\[
\begin{bmatrix}
17 & 12 & 18 \\
-16 & -9 & -24 \\
-5 & -4 & -4
\end{bmatrix}
\]

C :=

\[
\begin{bmatrix}
10 & -3 & -3 \\
-8 & 3 & 2 \\
-3 & 1 & 1
\end{bmatrix}
\]
a.) Now $A$ is represented with respect to the standard basis and the columns of $C$ form a basis for $\mathbb{R}^3$ (given). Find a matrix representation of $A$ with respect to the $C$ basis.