I. Define the following terms where \( A \) and \( B \) are subsets of the Reals.

1.) The set \( A \) is “bounded above”.
2.) \( x \) is a least upper bound of the set \( A \).
3.) A sequence of Real numbers.
4.) The limit of a sequence of real numbers.
5.) A Cauchy sequence.
6.) State, without proof, the Monotone Convergence and e Bolzano Weierstraus Theorem for sequences.
7.) When \( \Sigma a_n \) converges.

II. Prove or disprove

1.) If \( \{a_n\} \) and \( \{b_n\} \) are convergent sequences and if \( \{c_n\} \) is a sequence satisfying 
\[
a_n \leq c_n \leq b_n, \forall n \in \mathbb{N},
\]
then \( \{c_n\} \) converges.
2.) If \( \{a_n\} \) is a convergent sequence and \( \{b_n\} \) is a divergent sequence then \( \{a_n + b_n\} \) diverges.
3.) A bounded sequence converges.
4.) A convergent sequence is bounded.
5.) If \( x_n \to x \) and \( x_n \geq 0 \forall n \in \mathbb{N} \) then \( x \geq 0 \)
6.) If \( x_n \to x \) and \( x_n > 0 \forall n \in \mathbb{N} \) then \( x > 0 \)

III. Prove the following:

1.) Let \( \{x_n\} \) be a convergent sequence with \( \lim \{x_n\} = L \). Fix \( k \in \mathbb{N} \), and define a new sequence by \( y_n = x_{k+n} \). Using the definition of a limit to prove that \( \lim y_n = L \)

2.) Let \( x_1 \geq 2 \). Let \( x_{n+1} = 1 + \sqrt{x_n - 1} \) for \( n \geq 2 \)
   a.) prove that \( \{x_n\} \) is bounded below by 2.
   b.) prove that \( \{x_n\} \) is decreasing .
   c.) the \( x_n \) converges.
   d.) Find the limit.
3.) Let \( L \) be a lower bound for \( A \). Prove that \( L = \text{lub}A \) iff for every \( \epsilon > 0 \), there exists \( s_\epsilon \in A \) such that \( L - \epsilon < s_\epsilon \)
4.) Prove the existence or non-existence of limits for each of the following:
   i.) \( \{2n/(n + 4\sqrt{n})\} \)
   ii.) \( 1 + \{(−1)^n\} \)

IV. Use the definition of limit to prove each of the following:

1.) \( \lim \frac{4n + 3}{5n + 2} = \frac{4}{5} \)
2.) Let \( x_n \to L \) then \( \exists K \) such that \( \forall n > K, \frac{L}{2} < x_n < 3/2 \times L \)
V. Using limit theorems prove each of the following:

1.) \( \lim \frac{2^n}{n!} = 0 \)

2.) \( x_1 = 1 \) and \( x_{n+1} = \sqrt{3 + 2x_n}, n \geq 1 \) Prove that \( x_n \) converges and calculate the limit.

3.) \( \lim (2 + \frac{1}{n})^2 \)

4.) \( \lim (2 + \frac{1}{n})^n \)

5.) \( \lim \left( \frac{\cos(2n)}{n} \right) \)

6.) \( \lim \left( \frac{\sqrt{n} - 1}{\sqrt{n} + 1} \right) \)

V. Discuss the convergence / divergence of each of the following series:

1.) \( \lim \sum \frac{1}{n} \)

2.) \( \lim \sum \frac{1}{n!} \)

3.) \( \lim \sum \frac{1}{2^n} \)

4.) \( \lim \sum \frac{n^2}{n!} \)