1. i.) Find a finite collection $T$ (as small as possible) of $4 \times 4$ nilpotent matrices so that any nilpotent $4 \times 4$ with complex entries is similar to one of the matrices in the collection $T$. Justify.

ii.) Find two $7 \times 7$ nilpotent matrices with the same minimal polynomial $Z^3$ and same nullity (dim of kernel) but not similar. Justify.

2.) Let

$$A = \begin{pmatrix} 4 & 4 & -2 \\ 1 & 4 & -1 \\ 4 & 8 & -2 \end{pmatrix}.$$ 

i.) Find the characteristic polynomial of $A$.

ii.) Find the minimal polynomial of $A$.

iii.) Compute the matrix exponential $e^{At}$

iv.) Find $P$ such that $P^{-1}AP$ is in Jordan Form.

3.) Let

$$A = \begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}.$$ 

i.) Find the characteristic polynomial of $A$.

ii.) Find the minimal polynomial of $A$.

iii.) Compute the matrix square root of $A$ i.e. find $S$ such that $S^2 = A$.

4.) Let $T^2 = I$ is called a reflection.

i.) Find an example of a $2 \times 2$ reflection.

ii.) Find the eigenvalues of a reflection.

iii.) Prove that a reflection is diagonalizable.

5.) A matrix $A$ has Jordan form

$$J = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$ 

then $J = 2I + N$

i.) Show that $2I = p(J)$ and $N = q(J)$, where $p$ and $q$ are the polynomials:

$p(z) = (z^3 - 6z^2 + 12z)/4$ and $q(z) = z - p(z)$ $Hint : (A - 2I)^3 = 0$

6.) Do exercise 16,17 and 19, p. 96
7.) For each of the following matrices find the characteristic polynomial and the minimal polynomial.

i.)

\[ A = \begin{pmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix} \]

ii.)

\[ A = \begin{pmatrix} 5 & 0 & 0 & 0 & 0 \\ 0 & 5 & 1 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 7 \end{pmatrix} \]

iii.)

\[ A = \begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{pmatrix} \]

8.) Find the Jordan form for the following: (Hint: easy primary works)

i.)

\[ A = \begin{pmatrix} 1 & 2 & 0 & 0 \\ -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & -4 & 0 \end{pmatrix} \]