I. Do one and only one of the following:
1. 
   a. State and prove Schur’s lemma.
   b. State and prove an interesting consequence of Schur’s Lemma.

2. 
   a. State and prove the Singular value Decomposition Theorem.
   b. State and prove an interesting consequence of SVD.

II. Do one and only one of the following:
1. If \( c_1, c_2, c_3 \) are 3 distinct eigenvalues of an 3x3 matrix, prove that the corresponding eigenvectors are linearly independent.
2. Prove that if a 3x3 matrix has three linearly independent eigenvectors then the matrix is diagonalizable.

III.
1. Let \( A \) be a 3x3 matrix and \( \{ p_1, p_2, p_3 \} \) be a Jordan chain associated with the eigenvalue \( c \) where \( (A - c \times I)p_1 = 0 \) and \( (A - c \times I)p_i = p_{i-1} \) for \( i = 2, 3 \).
   a. What is the Jordan form.
   b. Prove directly that \( A \times P = J \times P \) where \( J \) is the respective Jordan form.

IV. Do the following:
   a. Prove that the eigenvalues of a nilpotent matrix are zero.
   b. Prove that the following matrix is nilpotent.

\[
N = \begin{pmatrix}
    1 & 1 & 1 \\
    -1 & -1 & -1 \\
    1 & 1 & 0
\end{pmatrix}
\]

V. Compute the following:
   a. Find the Jordan Form for:

\[
A = \begin{pmatrix}
    5 & -1 \\
    1 & 3
\end{pmatrix}
\]
   b. Find the Jordan Form for:

\[
A = \begin{pmatrix}
    2 & 1 \\
    1 & 2
\end{pmatrix}
\]