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Calculus I

FINAL EXAMINATION

MATH 585H

Name: _____

1. Consider the following functions of x where x is in the interval $[0, 2]$. Which one is <u>not</u> continuous at $x = 1$?

a) $|x - 1|$ **b)** $\frac{x^2-9}{x+3}$ **c)** $f(x) = \begin{cases} x - 3 & \text{if } x \geq 1 \\ x^2 - 2x - 1 & \text{if } x < 1 \end{cases}$ **d)** $\frac{(x-1)^2}{|x-1|}$

e) none of the above

2. Define P such that the following function is continuous at $x = 9$.

$$f(x) = \begin{cases} \frac{(x^2-81)}{4x-36} & \text{if } x \neq 9 \\ P & \text{if } x = 9 \end{cases}$$

a) 9 **b)** $\frac{9}{2}$ **c)** 0 **d)** 81 **e) none of the above**

3. Consider

$$f(x) = \begin{cases} -5x - 7 & \text{if } x < -1 \\ 7 & \text{if } x = -1 \\ -5x - 12 & \text{if } x > -1 \end{cases}$$

$\lim_{x\to -1^+} f(x)$ is:

a) 7 **b)** -2 **c)** -7 **d)** 0 **e) none of the above**

4. $\lim_{x\to -3^+} \frac{x^2+9}{x+3}$ is:

a) 3 **b)** 1 **c)** $-\infty$ **d)** $+\infty$ **e) none of the above**

5. $\lim_{x\to -3} \dfrac{2x^2 + 7x + 3}{3x^2 + 8x - 3}$ is:

a) -1 **b)** $\frac{2}{3}$ **c)** $\frac{1}{2}$ **d)** 0 **e) none of the above**

6. If $y = \cos^2(2x)$, then $D_x y$ when $x = \frac{\pi}{8}$ is:

a) -2 **b)** $\sqrt{2}$ **c)** $-\sqrt{2}$ **d)** 2 **e) none of the above**

7. **If** $y = \frac{1}{\sqrt[3]{x^2}}$**, then** $D_x y$ **when** $x = 1$ **is:**

a) 1 **b)** $-\frac{2}{3}$ **c)** -2 **d)** 2 **e) none of the above**

8. **If** $y = \frac{x^3}{4} - \frac{4}{x^3}$**, then** $D_x y$ **when** $x = 2$ **is:**

a) $\frac{3}{4}$ **b)** $-\frac{9}{4}$ **c)** $\frac{15}{4}$ **d)** $-\frac{13}{4}$ **e) none of the above**

9. **If** $x^4 - y^3 - 3x^2 = 99$**, then** $D_x y$ **is:**

a) $\frac{6x+4x^3}{3y^2}$ **b)** $4x^3 - 6x - 3y^2$ **c)** $\frac{6x-4x^3}{3y^2}$ **d)** $\frac{4x^3-6x}{3y^2}$ **e) none of the above**

10. **If** $y = \sin(x)\cos(2x)$**, then** $D_x y$ **when** $x = \frac{\pi}{4}$ **is:**

a) $\sqrt{2}/2$ **b)** $\sqrt{2}$ **c)** $-\sqrt{2}$ **d)** 0 **e) none of the above**

11. **If** $y = 3x^2\sqrt{4 - x^2}$**, then** $D_x y$ **when** $x = 1$ **is:**

a) $\frac{19}{\sqrt{3}}$ **b)** $5\sqrt{3}$ **c)** $2\sqrt{3}$ **d)** 0 **e) none of the above**

12. **If** $f'(x) = (x + 2)(x + 3)$**, then** f **is decreasing on:**

a) $[-3, -2]$ **only** **b)** $(-\infty, -3]$ **only** **c)** $(-\infty, +\infty)$

d) $[-1, +\infty)$ **only** **e) none of the above**

13. **If** $f''(x) = \frac{5x}{x+1}$ **then** f **is concave upward on:**

a) $(-\infty, 0)$ **and** $(1, +\infty)$ **b)** $(-\infty, -1)$ **and** $(0, +\infty)$ **c)** $(-\infty, -1)$ **only**
d) $(0, 1)$ **e) none of the above**

14. $f(x) = -4x^3 + 3x^2 + 18x + 77$ **has a relative maximum only when** x **is**

a) $\frac{3}{2}$ **and** -1 **b)** $\frac{3}{2}$ **c)** -1 **d)** 0 **e) none of the above**

15. **If** $y = 3x^{\frac{4}{3}}$**, then** $D_x^2 y$ **at** $x = 8$ **is:**

a) $-\frac{16}{3}$ **b)** $-\frac{64}{9}$ **c)** $\frac{1}{3}$ **d)** $\frac{16}{3}$ **e) none of the above**

16. **If** $f''(x) = x^2 - 4$**, then the function** f **has a point of inflection at** $x =$

a) -2 **and** 2 **only** **b)** $0, -2$**, and** 2 **c)** 2 **only** **d)** 0 **only** **e) none of the above**

17. $f(x) = 9x^4 + 8x^3 - 5$ **has a relative minimum at** $x =$

a) 0 **and** $-\frac{2}{3}$ **b)** 0 **only** **c)** $-\frac{2}{3}$ **only** **d)** $-\frac{4}{9}$ **only**
e) none of the above

18. **The graph of** $y = \frac{x-1}{x^2-4}$ **has**

a) $x = \pm 2$ **as horizontal asymptotes** **b)** $y = 0$ **as a horizontal asymptote**
c) $y = 1$ **as a vertical asymptote** **d)** $y = \pm 2$ **as horizontal asymptotes**
e) none of the above

19. The slope of the line tangent to $1 - xy = 2\sin y$ at the point $(0, \frac{\pi}{6})$ is:

a) undefined **b)** $-\frac{\pi}{(6\sqrt{3})}$ **c)** $\frac{(6-\pi)}{(6\sqrt{3})}$ **d)** 0 **e) none of the above**

20. If 1200 squae cm of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

Show all work!

21. A spherical ball of ice with an initial radius of 4 inches melts at a rate of 2 in^3 per minute. How fast is the radius of the ball decreasing when the radius is 3 inches? $(V = \frac{4}{3}\pi r^3)$

a) $\frac{1}{36\pi}\frac{in.}{min}$ **b)** $18\pi\frac{in.}{min}$ **c)** $72\pi\frac{in.}{min}$ **d)** $\frac{1}{18\pi}\frac{in.}{min}$
e) none of the above

22. $\int_0^1 4x(x^2+2)^3 dx$ **is:**

a) 65 **b)** $\frac{65}{2}$ **c)** 130 **d)** $\frac{81}{2}$ **e) none of the above**

23. $\int_0^1 x^3\sqrt{x^4+1}\, dx$ **is:**

(a) $\frac{2\sqrt{2}-1}{24}$ **(b)** $\frac{4\sqrt{2}-2}{3}$ **(c)** $\frac{\sqrt{2}}{3}$ **(d)** $\frac{2\sqrt{2}-1}{6}$
(e) None of the above.

24. **The solution of** $y' = 2x + 3$ **satisfying** $y = 2$ **when** $x = 0$ **is**

(a) $y = x^2 + 3x + 2$ **(b)** $y = 2$

(c) $y = x^2 + 3x$ **(d)** $y = 2x^2 + 3x + 2$

(e) None of the above.

25. **The solution to** $y' = 2x^2 + 6x + 2$ **satisfying** $y = 1$ **when** $x = 0$ **is:**

a) $\frac{2x^3}{3} + 3x^2 + 2x$ **b)** $\frac{2x^3}{3} + 3x^2 + 2x - \frac{17}{3}$

c) $\frac{2x^3}{3} + 3x^2 + 2x + 1$ **d)** $\frac{2x^3}{3} + 3x^2 + 3$

e) none of the above

26. **The area bounded by the curves** $y = 2 - x^2$ **and** $x + y = 0$ **is:**

a) $\frac{10}{3}$ **b)** $\frac{4\sqrt{2}}{3}$ **c)** $\frac{3}{2}$ **d)** $\frac{9}{2}$ **e) none of the above**

27. **The area bounded by the curves** $y = x + 2,\ y = x^2$ **and to the right of the** y**-axis is**

(a) 6 **(b)** 4 **(c)** $\frac{10}{3}$ **(d)** $\frac{1}{3}$

(e) None of the above.

28. $\int_3^1 \frac{dx}{(x+1)^3}$ **is:**

a) $\frac{1}{8}$ **b)** $\frac{3}{16}$ **c)** $-\frac{3}{32}$ **d)** $-\frac{3}{16}$ **e) none of the above**

29. $\int_0^2 x^2\sqrt{2x^3+1}dx$ **is:**

a) $\frac{1}{6}((17)^{(\frac{3}{2})} - 1)$ **b)** $4((17)^{(\frac{1}{2})} - 1)$ **c)** $\frac{1}{9}((17)^{(\frac{3}{2})} - 1)$

d) $96((17)^{(-\frac{1}{2})} - 1)$ **e) none of the above**

30. $D_x \int_4^x \sqrt{t^2-7}dt$ **is:**

a) $\frac{2}{3}(x^2-7)^{\frac{3}{2}}$ **b)** $\sqrt{t^2-7}$ **c)** $\sqrt{x^2-7}-3$ **d)** $\sqrt{x^2-7}$

e) none of the above

31. $\int_0^1 (x^2+1)^7 x\ dx$ **is:**

(a) $\frac{21}{3}$ **(b)** $\frac{255}{8}$ **(c)** $\frac{255}{2}$ **(d)** $\frac{255}{16}$

(e) None of the above.

32. $\int_{-7}^{-2} \sqrt{2-x}\ dx$ **is:**

(a) $-\frac{38}{3}$ **(b)** $\frac{70}{3}$ **(c)** 18 **(d)** $\frac{38}{3}$

(e) None of the above.

33. The volume of the solid generated by revolving about the x-axis the region bounded by the graphs of $y = x^3, x = 2$, and the x-axis is described by

(a) $\int_0^8 \pi x^6\ dx$ **(b)** $\int_0^8 2\pi x^4\ dx$

(c) $\int_0^2 \pi x^6\ dx$ **(d)** $\int_0^2 2\pi x^4\ dx$

(e) None of the above.

34. The volume of the solid generated by revolving about the x-axis the region bounded by the graphs of $y = \sqrt{x},\ x = 0$, and $y = 2$ is described by

(a) $\int_0^2 \pi y^3\ dy$ **(b)** $\int_0^2 \pi y^2\ dy$ **(c)** $\int_0^2 2\pi y^3\ dy$ **(d)** $\int_0^2 \pi y^4\ dy$

(e) None of the above.

B1. $\lim_{h\to 0} \frac{\sin(\pi+h)-\sin\pi}{h}$ **is:**

a) 1 **b)** -1 **c)** 0 **d)** $+\infty$ **e) none of the above**

B2. If $f'(x) = (x+1)(x+2)(x+3)$, then f is increasing on:

a) $[-3,-2]$ **only** **b)** $(-\infty,-3]$ **only** **c)** $(-\infty,+\infty)$

d) $[-1,+\infty)$ **only** **e) none of the above**

B3. A box with an open top is to be constructed from a 15" by 24" piece of cardboard by removing equal size squares from each corner and folding the resulting flaps upward. The box with the largest volume will have a height of (Hint: $12x30 = 360, 12x13 = 156$**.)**

a) $3in$ **b)** $\frac{7}{2}in$ **c)** $4in$ **d)** $5in$ **e) none of the above**

itemB4. $\int_{-11}^{-4} \sqrt{5-x}dx$ **is:**

a) $\frac{182}{3}$ **b)** $\frac{74}{3}$ **c)** $-\frac{128}{3}$ **d)** $\frac{128}{3}$ **e) none of the above**

B5. The area bounded by the curves $y = 3x+2,\ y = x^3$ **and to the right of the** y**-axis is**

(a) 6 **(b)** 4 **(c)** $\frac{27}{4}$ **(d)** $\frac{21}{2}$

(e) None of the above.

B6. If $y = \frac{1}{x} - \frac{1}{\log(x)}$**, then** $D_x y$ **is:**

a) $\frac{x-(log(x))^2}{(xlog(x))^2}$ **b)** $\frac{x^2-2log(x)}{(2x^3log(x))}$ **c)** $\frac{x-1}{(xlog(x))^2}$ **d)** $\frac{2x-1}{(xlog(x))^2}$ **e) none of the above**

34. $\int_0^1 \frac{2\ dx}{(x+1)^3}$ **is:**

(a) $\frac{1}{2}$ **(b)** $\frac{1}{4}$ **(c)** $\frac{3}{4}$ **(d)** $\frac{4}{3}$

(e) None of the above.

B7. $D_x \int_4^x \sqrt{t^2-7}dt$ **is:**

a) $\frac{2}{3}(x^2-7)^{\frac{3}{2}}$ **b)** $\sqrt{t^2-7}$ **c)** $\sqrt{x^2-7}-3$ **d)** $\sqrt{x^2-7}$

e) none of the above

B8. $\lim_{x\to 0} \frac{sqrt(x+1)-1}{x}$ **is:**

a) $-1/2$ **b)** $\frac{1}{2}$ **c)** 1 **d)** 2 **e) none of the above**

B9. Let a **be the smallest positive value of** x **at which the function** $f(x) = (\cos(x^2)(\sin(x^2)$ **has a crtitical point. What is thee value of** $f(a)$**.**

a) $\sqrt{(}2)/4$ **b)** $1/2$ **c)** $\sqrt{(}2)/2$ **d)** $\sqrt{(}2)$ **e) none of the above**

B10. If $\int_0^b \tan(x)dx = 2$ **, then** b **could equal**

a) $\cos^{-1}(2e)$ **b)** $\sec^{-1}(2)$ **c)** $(\sec^{-1}(e))^2$ **d)** $\sec^{-1}(e^2)$ **e) none of the above**

B11. The solution to $y' = \frac{2x}{y^2-1}$ **passes the point** $(2,3)$ **is:**

a) $x^2-2 = \frac{1}{3}y^3+y$ **b)** $x^2-1 = \frac{1}{3}y^3-2y$

c) $x^2+1 = \frac{1}{3}y^3+y$ **d)** $x^2+2 = \frac{1}{3}y^3-y$

e) none of the above

B12. **If one arch of the curve $y = \sin(x)$ is revolved around the $x - axis$, what is the volume of the generated solid.**

(a) $\frac{\pi^2}{4}$ **(b)** $\frac{\pi^2}{2}$ **(c)** 2π **(d)** π^2

(e) None of the above.

B13. **If a is a positive constant, what is the maximum value of $f(x) = \frac{log(x)}{x^a}$**

a) $\frac{1}{a*e}$ **b)** a^e **c)** e^a **d)** $\frac{e^{a^2}}{a}$ **e) none of the above**

B14. $1 + \cot^2(\sin -1(\pi 4) =$

a) $\frac{3}{2}$ **b)** 2 **c)** 3 **d)** $\frac{16}{\pi^2}$ **e) none of the above**

B15. **What is the maximim value of $f(x) = 2x^3 + 12x^2 - 30x + 10$ on the interval $x \leq 0$**

a) -210 **b)** -36 **c)** -5 **d)** 210 **e) none of the above**

B16. $\lim_{x\to 2} \frac{x^2-4}{x-2}$ **is:**

a) 0 **b)** 2 **c)** 4 **d)** $+\infty$ **e) none of the above**

B17. **The slope of the line tangent to a circle centered at the origin with a radius of** 5 **at the point** $(3, 4)$ **is:**

a) $\frac{3}{4}$ **b)** $-\frac{4}{3)}$ **c)** $\frac{4}{3}$ **d)** $-\frac{3}{4}$ **e) none of the above**

B18. **If $y = e^{-x}\sin(2x)$, then $D_x y$ is:**

a) $e^{-x}(2\cos(2x) - \sin(2x))$ **b)** $-e^{-x}(2\sin(2x) + \cos(2x))$ **c)** $e^{-x}(2\sin(2x) + \cos(2x))$ **d)** $-e^{-x}(2\cos(2x) - \sin(2x))$ **e) none of the above**

B19. $\lim_{x\to\pi} \frac{x^2-\pi x+\sin(x)}{-\sin(x)}$ **is:**

a) 0 **b)** 1 **c)** $\pi - 1$ **d)** $2 * (\pi - 1)$ **e) none of the above**

B20. **Determine** $\int \frac{x^3 + x + 4}{x^2} dx$ **is:**

a) $\frac{x}{4} + \ln|x| - \frac{4}{x} + C$ **b)** $-\frac{x}{2} + \ln(x) - 8x + C$ **c)** $\frac{x^2}{2} + \ln|x| - \frac{2}{x^2} + C$ **d)** $\frac{x^2}{4} + \ln|x| - \frac{4}{x} + C$ **e) none of the above**

B21. **The area bounded by the lines $y = x$, $x + 2y = 6$ and the x $axis$ is:**

a) 5 **b)** 5.5 **c)**6 **d)** 6.5 **e) none of the above**

B22. **The area bounded by the curve $y = \frac{1}{x}$, $y = 2$,$y = 10$ and the y $axis$ is approximately:**

a) $.48$ **b)** 1.6 **c)**2.1 **d)** 3.0 **e)** 4

B23. The area bounded by the curve $y = 8 - x^2$, $y = -2 + x^2$ is approximately:

a) 22.4 **b)** 26.8 **c)**29.8 **d)** 44.7.0 **e)** 46.1

B24. The area bounded by the curve $y = \sin(x)$, $y = 0$ over the interval from $0 \leq x \leq 2 * \pi$ is:

a) 0 **b)** $\frac{\pi}{2}$ **c)**2 **d)** 4 **e) none of the above**

B25. What is the slope of the curve $y = 10x^2 - 3x - 1$ as it crosses the positive part of the $x - axis$.

a) $\frac{3}{20}$ **b)** $\frac{1}{5}$ **c)**$\frac{1}{3}$ **d)** 7 **e) none of the above**

B26. What is the volume of revolution from $x = 0$ to $x = \frac{3}{2}$ when the function $f(x) = 2x^2$ is revolved around the $y - axis$.

a) $\frac{3\pi}{2}$ **b)** $\frac{27\pi}{16}$ **c)**$\frac{35\pi}{16}$ **d)** $\frac{81\pi}{16}$ **e) none of the above**

B26. What is the lendth of the line with slope $\frac{4}{3}$, from the point $(6, 4)$ to the $y - axis$.

a) 10 **b)** 25 **c)**50 **d)** 75 **e) none of the above**

B27. A line goes through the point $(4, -6)$ and is perpendicular to the line $y = 4x = 10$. What is the equation of the line?

a) $y = 4x - 20$ **b)** $y = -\frac{x}{4} - 5$ **c)**$y = \frac{x}{5} + 5$ **d)** $y = \frac{x}{4} + 5$ **e) none of the above**

B28. What is the length of the curve $y = \sqrt{(}x^3)$ from $x = 0$ to $x = 28$?

a) 9 **b)** $\frac{x}{408}$ **c)**45 **d)** $\frac{4088}{27}$ **e) none of the above**

B29. The readius of a circle is decreasing at a rate of 0.5 cm per second. At what rate, in cm^2 per second, is the circle's area decreasing when the radius is 4 cm?

a) 4π **b)** 2π **c)** π **d)** $\pi/2$ **e)** $\pi/4$

B30. Define P such that the following function is continuous at $x = 2$.

$$f(x) = \begin{cases} \frac{(x^2-6x+8)}{x^3-2x^2+2x-4} & \text{if } x \neq 2 \\ P & \text{if } x = 2 \end{cases}$$

a) -1 **b)** $\frac{1}{2}$ **c)** $-\frac{1}{2}$ **d)** $-\frac{1}{3}0$ **e) none of the above**

B31. What is the equation of the tangent line to the curve $y = x^3 - 3x^2 + 4x$ at the curve's point of inflection? a) $y - 2x - 3$ **b)** $y = x - 1$ **c)** $y = x + 1$ **d)** $y = 3x - 2$ **e)** $x + y = 1$