```
> restart:with( plots):with (student):
Warning, the name changecoords has been redefined
```

THESE ARE THE SOLUTIONS TO THE SAMPLE EXAM QUESTIONS

page 1 Section I ex 1

```
> g(x):= sin(3*x)/x:
> Limit(g(x), x=0)=limit(g(x), x=0);
```

$$\lim_{x \to 0} \frac{\sin(3\,x)}{x} = 3$$

ex2

```
> g(x):= (x^2-1)/(x-1):
> Limit(g(x), x=1)=limit(g(x), x=1);
```

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2$$

ex3

```
> f(x):= sqrt(x):
> Limit((sqrt(x+h)-sqrt(x))/h, h=0)=limit((sqrt(x+h)-sqrt(x))/h,
  h=0);
```

$$\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{1}{2}\frac{1}{\sqrt{x}}$$

ex4

```
> g(x):= (2*x^2 + 7*x + 3)/(3*x^2+8*x-3):
> Limit(g(x), x=infinity)=limit(g(x), x=infinity);
```

$$\lim_{x \to \infty} \frac{2\,x^2 + 7\,x + 3}{3\,x^2 + 8\,x - 3} = \frac{2}{3}$$

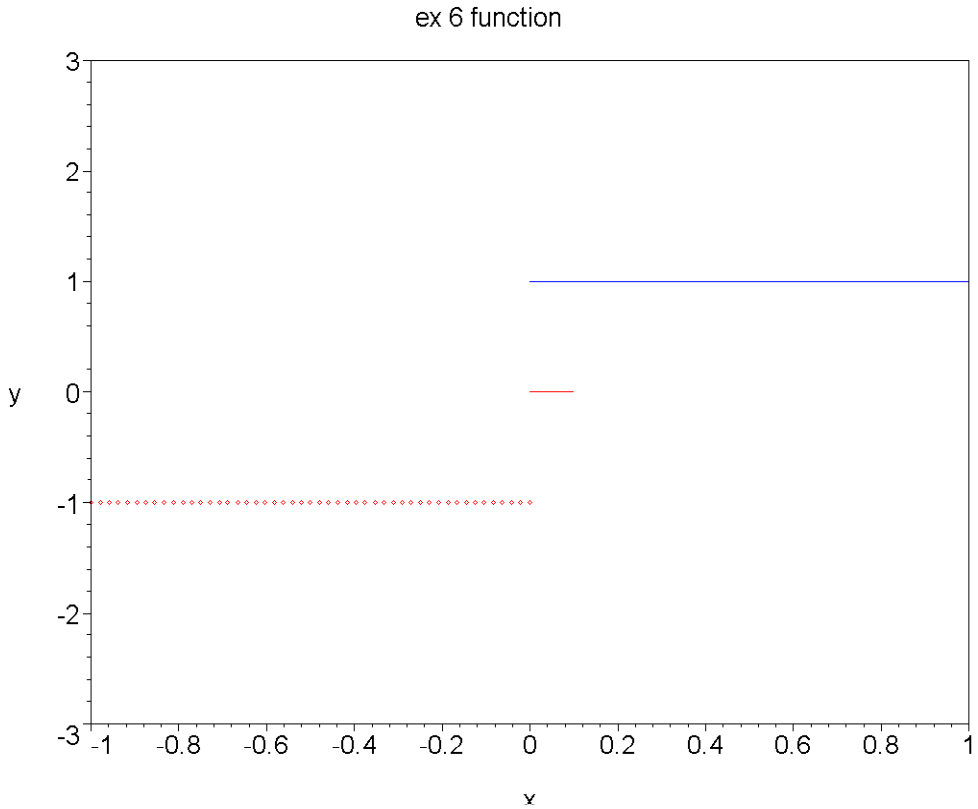
ex5

```
> g(x):= (cos(x)/x):
> Limit(g(x), x=infinity)=limit(g(x), x=infinity);
```

$$\lim_{x \to \infty} \frac{\cos(x)}{x} = 0$$

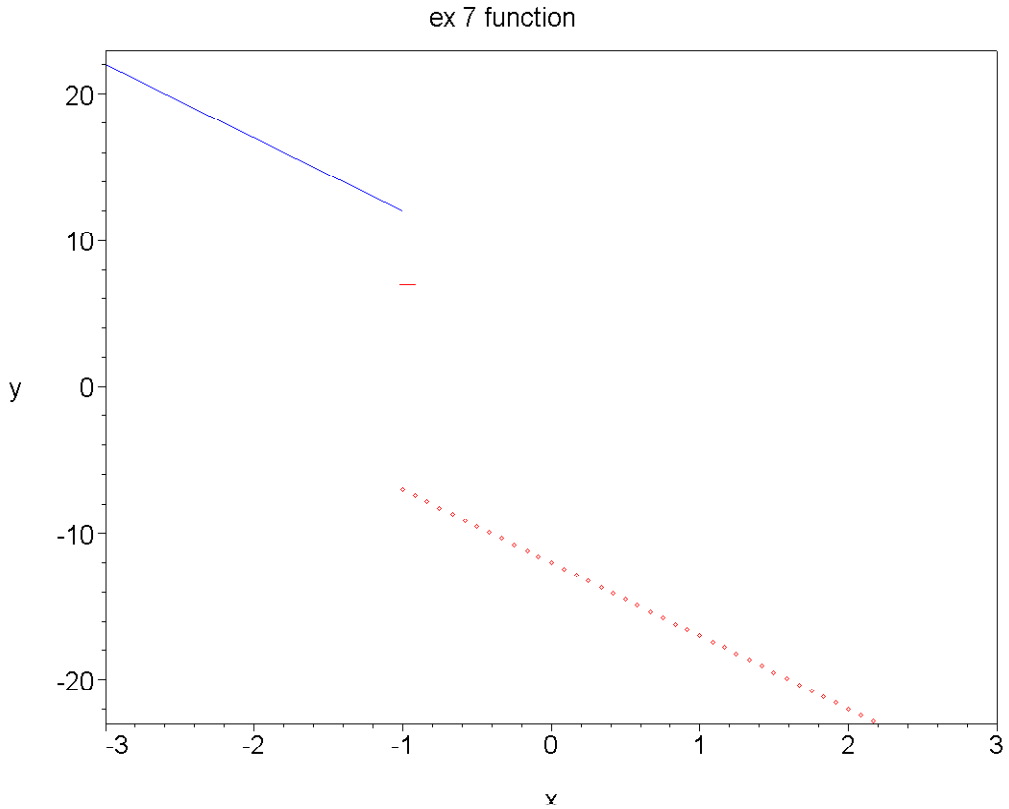
Section II ex6

```
> F:= plot(1,x=0..1,y=-3..3,style=line,color=blue): G:=
  plot(-1,x=-1..0,y=-3..3,style=point,color= red):H:= plot(0,x=0..
  .1,y=-3..3,style=line):
  display({F,G,H},axes=boxed,title=`ex 6 function `);
```



Section II ex7

```
> F:= plot(-5*x+7,x=-3..-1,y=-23..23,style=line,color=blue): G:=
  plot(-5*x-12,x=-1..3,y=-23..23,style=point,color= red):H:=
  plot(7,x=-1.02.. -.91,y=-3..3,style=line):
  display({F,G,H},axes=boxed,title=`ex 7 function `);
```

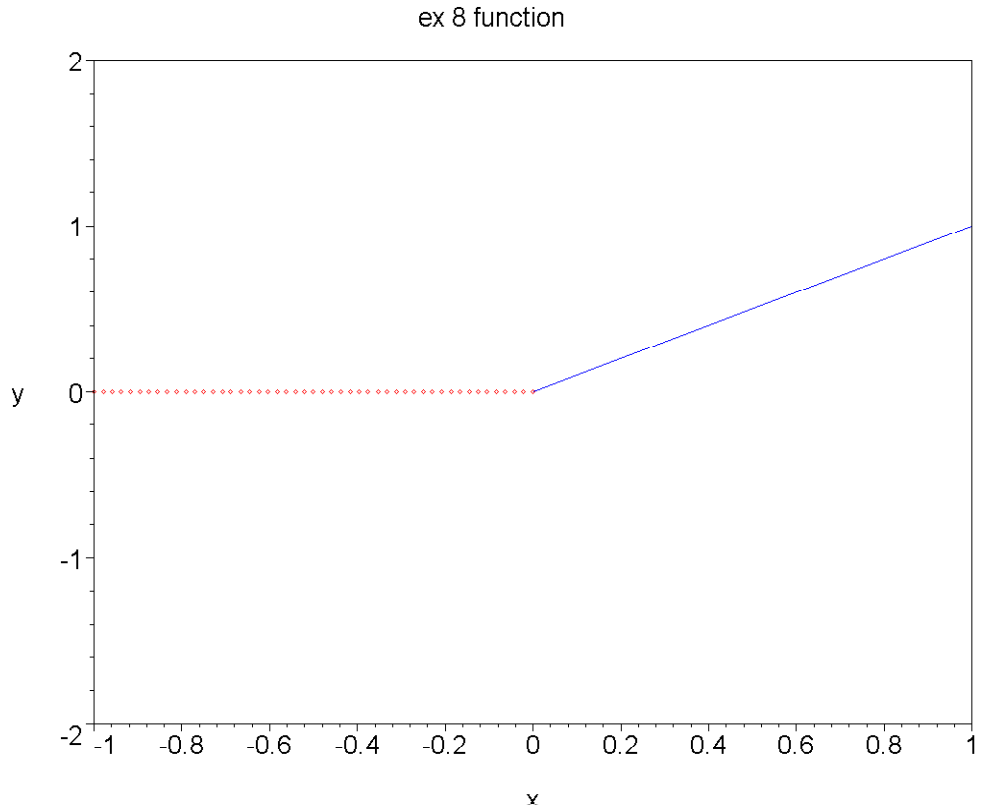


Section III ex8

```
> g(x):= (x^2)/(abs(x));
> Limit(g(x), x=0)=limit(g(x), x=0);G:=
  plot(0,x=-1..0,y=-2..2,style=point,color= red):
> F:= plot(x^2/(abs(x)),x=.001..1,y=-2..2,style=line,color=blue):
  display({F,G},axes=boxed,title=`ex 8 function `);
```

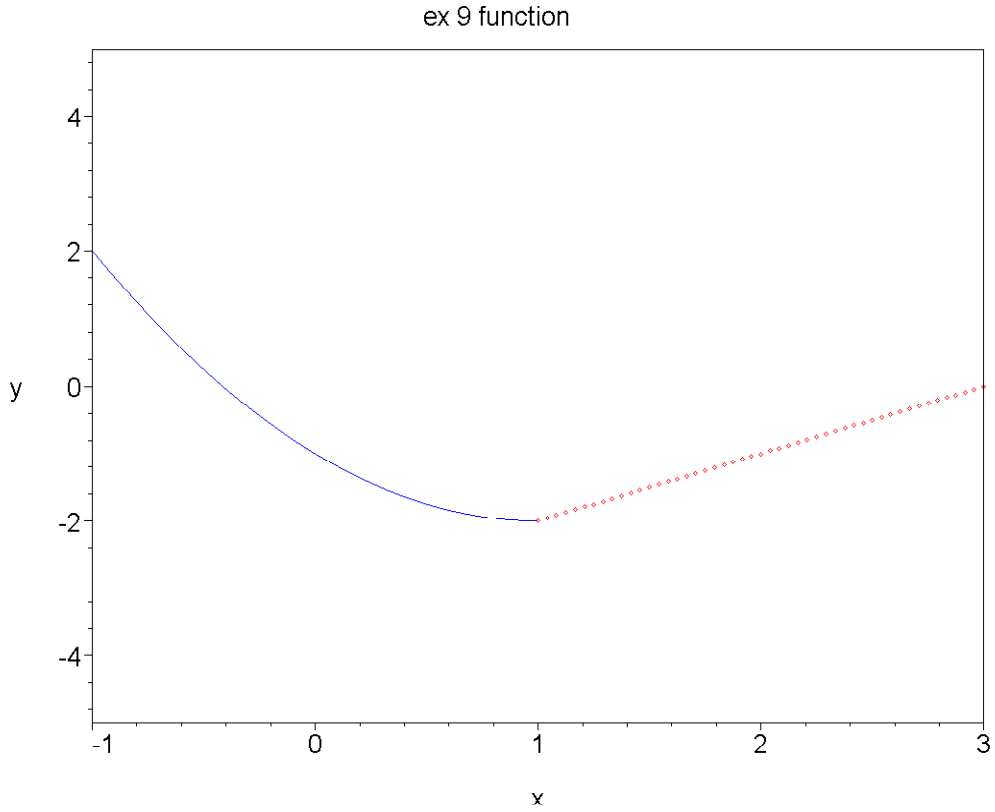
$$g(x) := \frac{x^2}{|x|}$$

$$\lim_{x \to 0} \frac{x^2}{|x|} = 0$$



Section III ex9

```
> G:= plot(x-3,x=1..3,y=-5..5,style=point,color= red):
> F:= plot(x^2-2*x-1,x=-1..1,y=-5..5,style=line,color=blue):
  display({F,G},axes=boxed,title=`ex 9 function `);
```



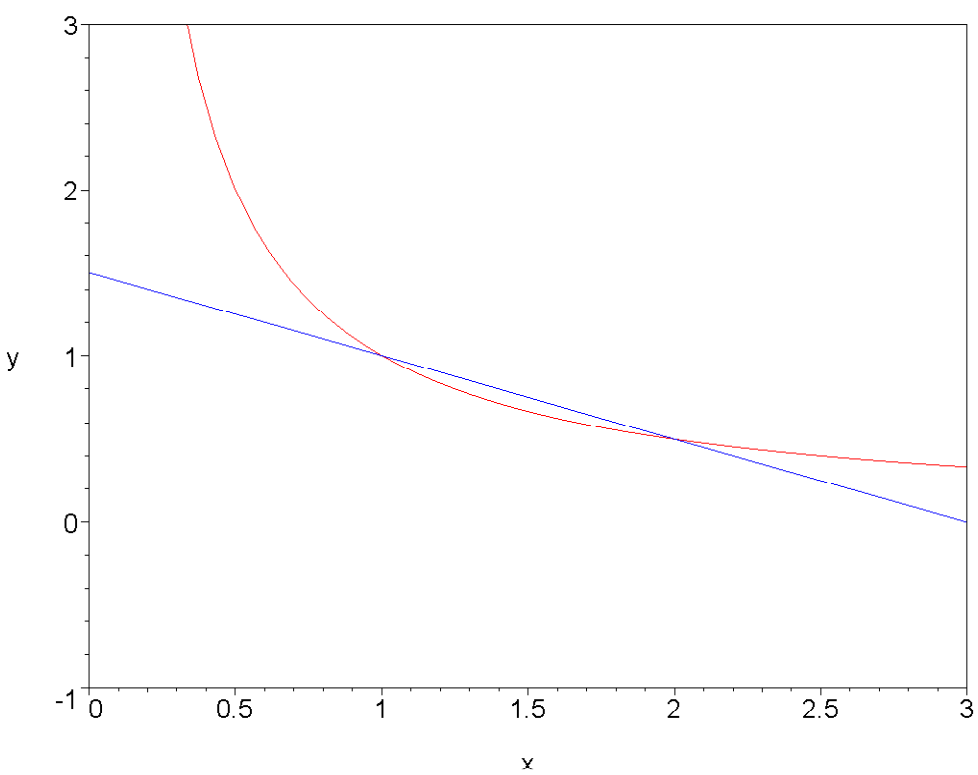
Section IV ex10

```
> g(x):= 1/x:m:= 1/2-1/1; seline(x):=m*(x-2)+1/2;
```

$$m := \frac{-1}{2}$$

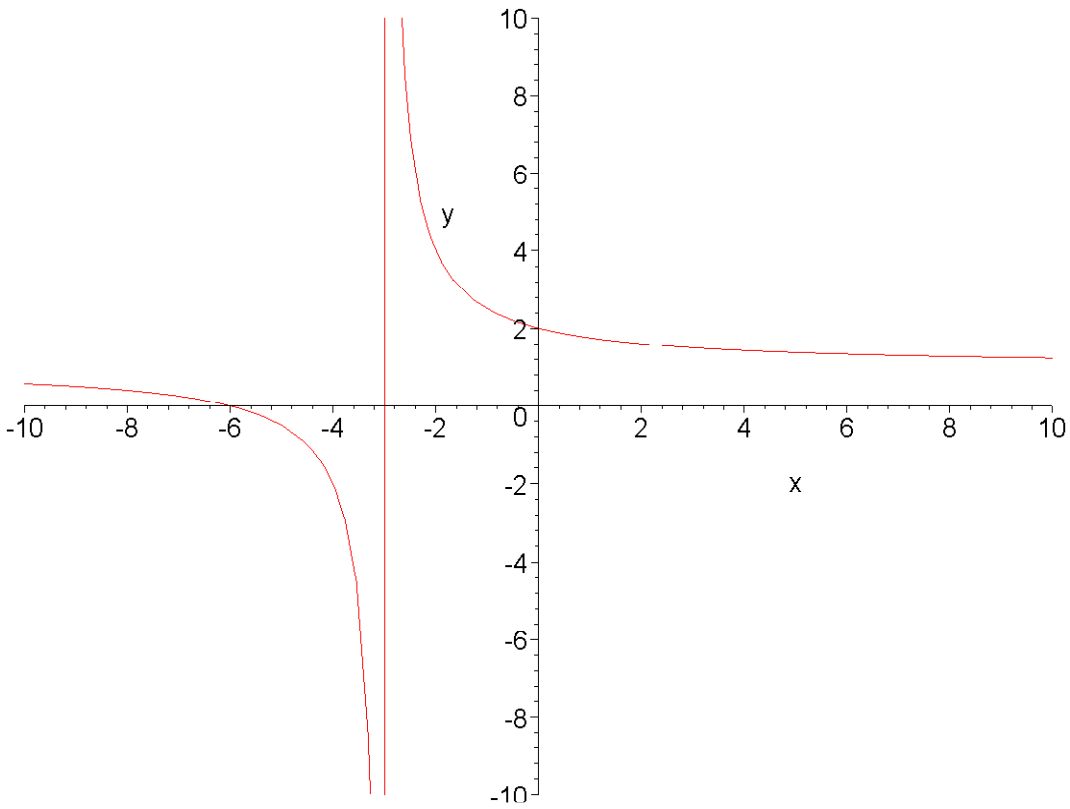
$$\text{seline}(x) := -\frac{1}{2}x + \frac{3}{2}$$

```
> G:= plot(g(x),x=0..3,y=-1..3,color= red):
> F:= plot(seline(x),x=0..3,y=-1..3,style=line,color=blue):
  display({F,G},axes=boxed,title=`function and secant line `);
```



Section V ex11

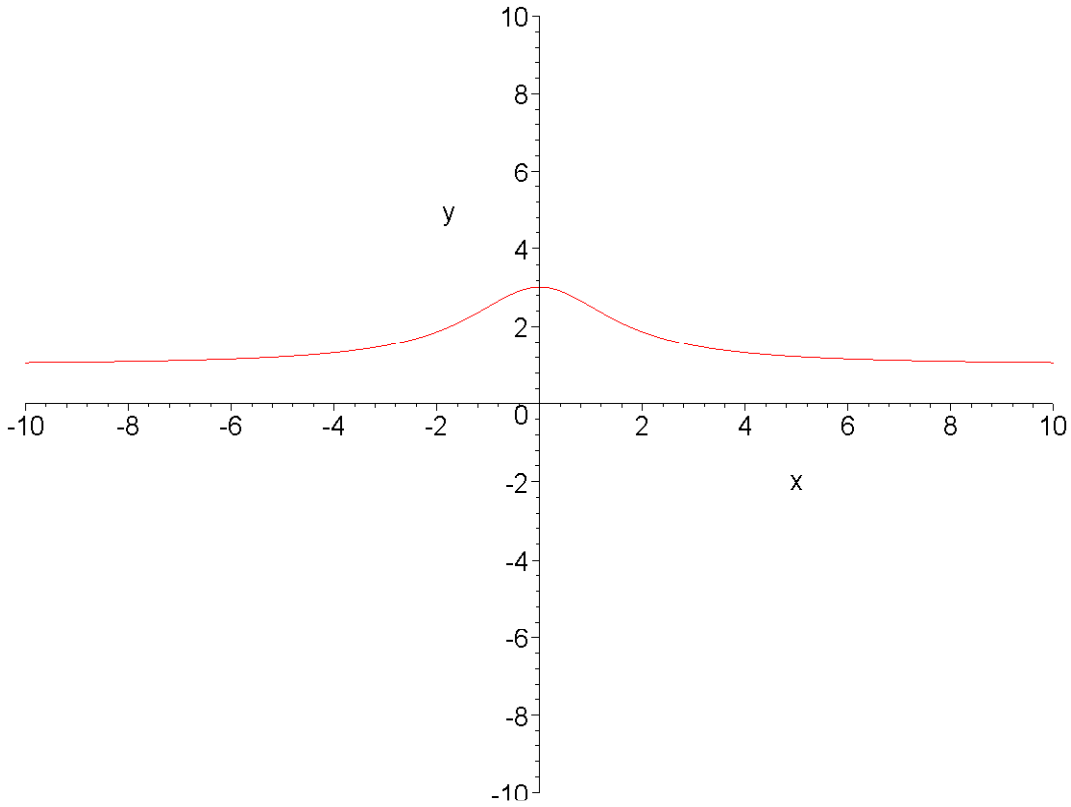
```
> plot((x+6)/(x+3),x= -10 .. 10, y=-10..10);
```



ex12

```
> g(x):= (x^2+9)/(x^2+3):
> Limit(g(x), x=infinity)=limit(g(x), x=infinity);
> plot(g(x),x= -10 .. 10, y=-10..10);
```

$$\lim_{x \to \infty} \frac{x^2 + 9}{x^2 + 3} = 1$$



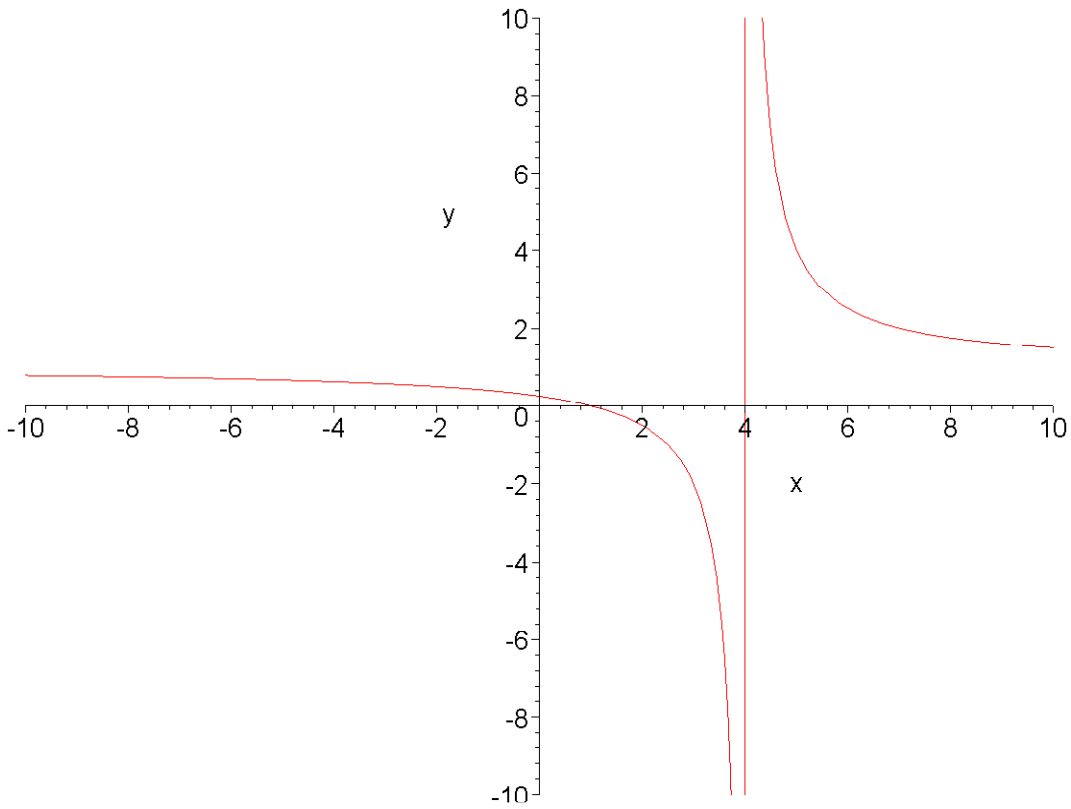
ex 13

```
> g(x):= (cos(x)-1)/x:
> Limit(g(x), x=0)=limit(g(x), x=0);
```

$$\lim_{x \to 0} \frac{\cos(x) - 1}{x} = 0$$

Page 2 ex 21

```
> plot((x-1)/(x-4),x= -10 .. 10, y=-10..10);
```



ex 17

```
> Limit(((cos(x+h))/(x+h)- (cos(x))/x)/h,
  h=0)=limit(((cos(x+h))/(x+h)- (cos(x))/x)/h, h=0);eval(%,x=Pi/8);
```

$$\lim_{h \to 0} \frac{\frac{\cos(x+h)}{x+h} - \frac{\cos(x)}{x}}{h} = -\frac{\sin(x)\,x + \cos(x)}{x^2}$$

$$\lim_{h \to 0} \frac{\frac{\cos\left(\frac{1}{8}\pi + h\right)}{\frac{1}{8}\pi + h} - \frac{8\cos\left(\frac{1}{8}\pi\right)}{\pi}}{h} = -64\,\frac{\frac{1}{8}\sin\left(\frac{1}{8}\pi\right)\pi + \cos\left(\frac{1}{8}\pi\right)}{\pi^2}$$

ex 25

```
> g(x):= (9-x)/(3-sqrt(x)):
> Limit(g(x), x=infinity)=limit(g(x), x=infinity);
> Limit(g(x), x= - infinity)=limit(g(x), x= -infinity);
```

$$\lim_{x \to \infty} \frac{9-x}{3-\sqrt{x}} = \infty$$

$$\lim_{x \to (-\infty)} \frac{9-x}{3-\sqrt{x}} = \infty\, I$$

ex22

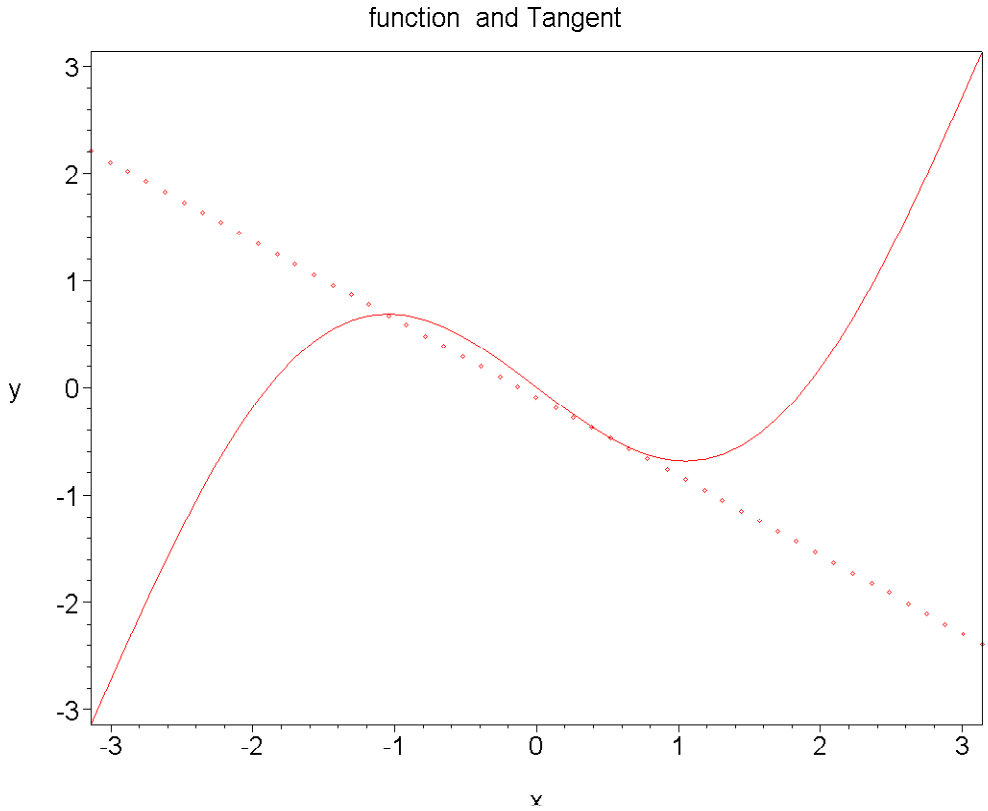
```
> g:= x -> x - 2*sin(x):
  Diff(g(x),x)= simplify(diff(g(x),x));eval(%,x=Pi/6);m:=1-sqrt(3);
```

$$\frac{\partial}{\partial x}(x - 2\sin(x)) = 1 - 2\cos(x)$$

$$\left(\frac{\partial}{\partial x}(x - 2\sin(x))\right)\Big|_{x = 1/6\,\pi} = 1 - \sqrt{3}$$

$$m := 1 - \sqrt{3}$$

```
> m := 1-sqrt(3):lin := x -> m*x -m*Pi/6+g(Pi/6):
  F:= plot(x-2*sin(x),x=-Pi..Pi,y=-Pi..Pi,style=line): G:=
  plot(lin(x),x=-Pi..Pi,y=-Pi..Pi,style=point):
  display({F,G},axes=boxed,title=`function  and Tangent`);
```



ex 18

```
> g:= x -> 1/x^(-2/3):
  Limit((g(x+h)- g(x))/h, h=0)=limit((g(x+h)- g(x))/h,
  h=0);eval(%,x=1);
```

$$\lim_{h \to 0} \frac{(x+h)^{(2/3)} - x^{(2/3)}}{h} = \frac{2}{3}\frac{1}{x^{(1/3)}}$$

$$\lim_{h \to 0} \frac{(1+h)^{(2/3)} - 1}{h} = \frac{2}{3}$$

ex 15

```
> g(x):= (x^2-4)/(x-2):
> Limit(g(x), x=3, right)=limit(g(x), x=3,right);
```

$$\lim_{x \to 3+} \frac{x^2-4}{x-2} = 5$$

ex 16

```
> g(x):= (2*x^2 +7*x+3)/(3*x^2+8*x-3):
> Limit(g(x), x=-3)=limit(g(x), x=-3);
```

$$\lim_{x \to (-3)} \frac{2x^2+7x+3}{3x^2+8x-3} = \frac{1}{2}$$

still page 2 comput the following ...

ex1

```
> g(x):= (x^2 - 4)/(x-2):
> Limit(g(x), x=2)=limit(g(x), x=2);
```

$$\lim_{x \to 2} \frac{x^2-4}{x-2} = 4$$

ex2

```
> g(x):= (x - 4)/(2-sqrt(x)):
> Limit(g(x), x=4)=limit(g(x), x=4);
```

ex3

```
> g:= x -> 1/x:
  Limit((g(x+h)- g(x))/h, h=0)=limit((g(x+h)- g(x))/h, h=0);
```

$$\lim_{x \to 4} \frac{x-4}{2-\sqrt{x}} = -4$$

$$\lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = -\frac{1}{x^2}$$

ex 6

```
> g:= x -> (2*sin(3*x))/x:
  Limit( g(x), x=infinity)=limit( g(x), x=infinity);
>
```

$$\lim_{x \to \infty} 2\,\frac{\sin(3\,x)}{x} = 0$$

ex7

```
> g:= x -> (3-sqrt(x))/(9-x):
  Limit( g(x), x=infinity)=limit( g(x), x=infinity);
>
```

$$\lim_{x \to \infty} \frac{3-\sqrt{x}}{9-x} = 0$$

ex 8

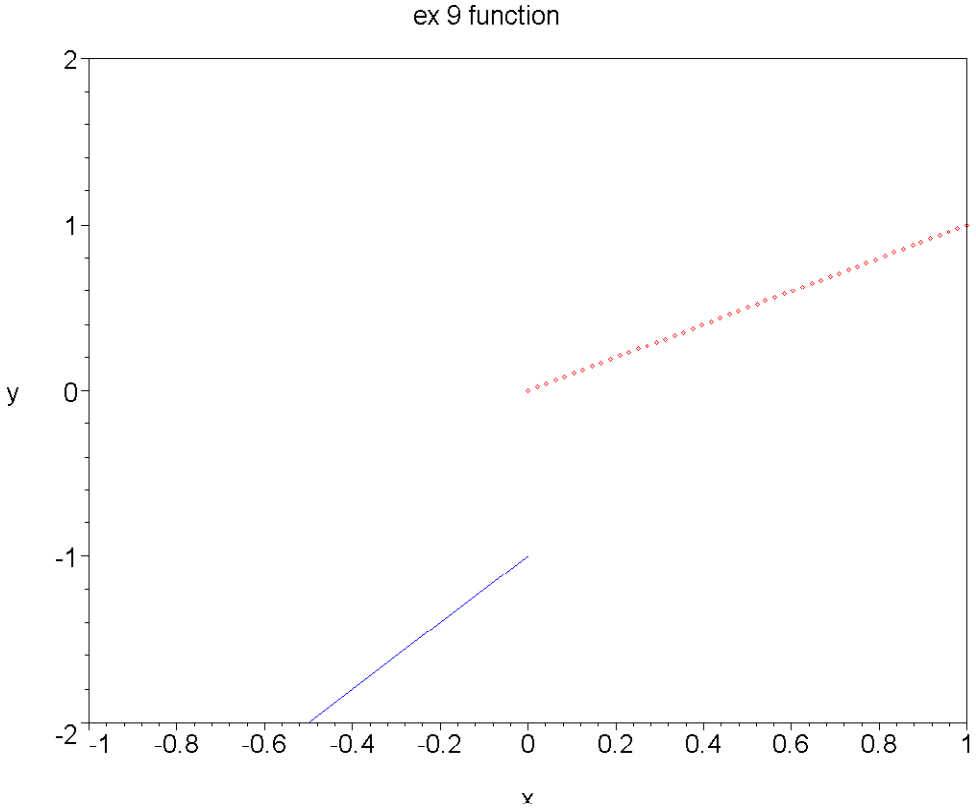
```
> g:= x -> sqrt(x)+1;m:= eval( (g(2)-g(1))/(2-1)): secantline:= x ->
  m*(x-2)+g(2);
>
```

$$g := x \to \sqrt{x} + 1$$

$$secantline := x \to m\,(x-2) + g(2)$$

Page 3 ex9

```
> G:= plot(x,x=0..1,y=-2..2,style=point,color= red):
> F:= plot(2*x - 1,x=-1..0,y=-2..2,style=line,color=blue):
  display({F,G},axes=boxed,title=`ex 9 function `);
```



ex10

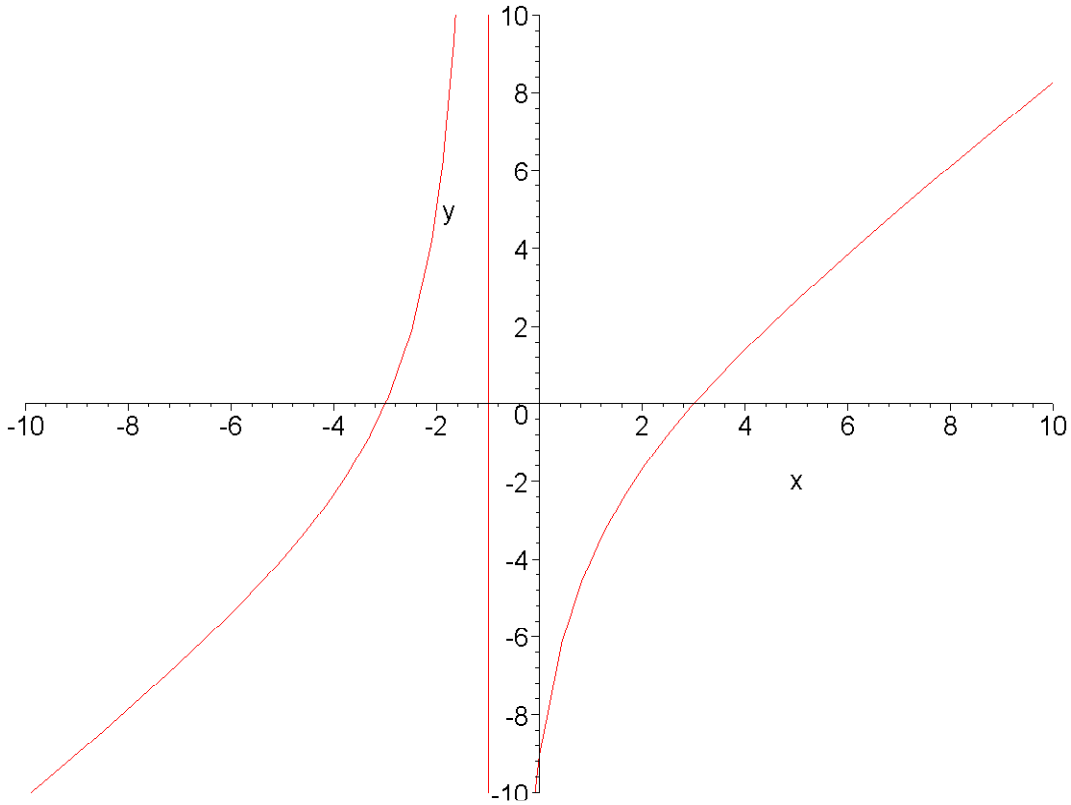
```
> g:= x -> (x^2-81)/(4*x-36);
  Limit(g(x), x=9)=limit( g(x), x=9);
```

$$g := x \to \frac{x^2 - 81}{4x - 36}$$

$$\lim_{x \to 9} \frac{x^2 - 81}{4x - 36} = \frac{9}{2}$$

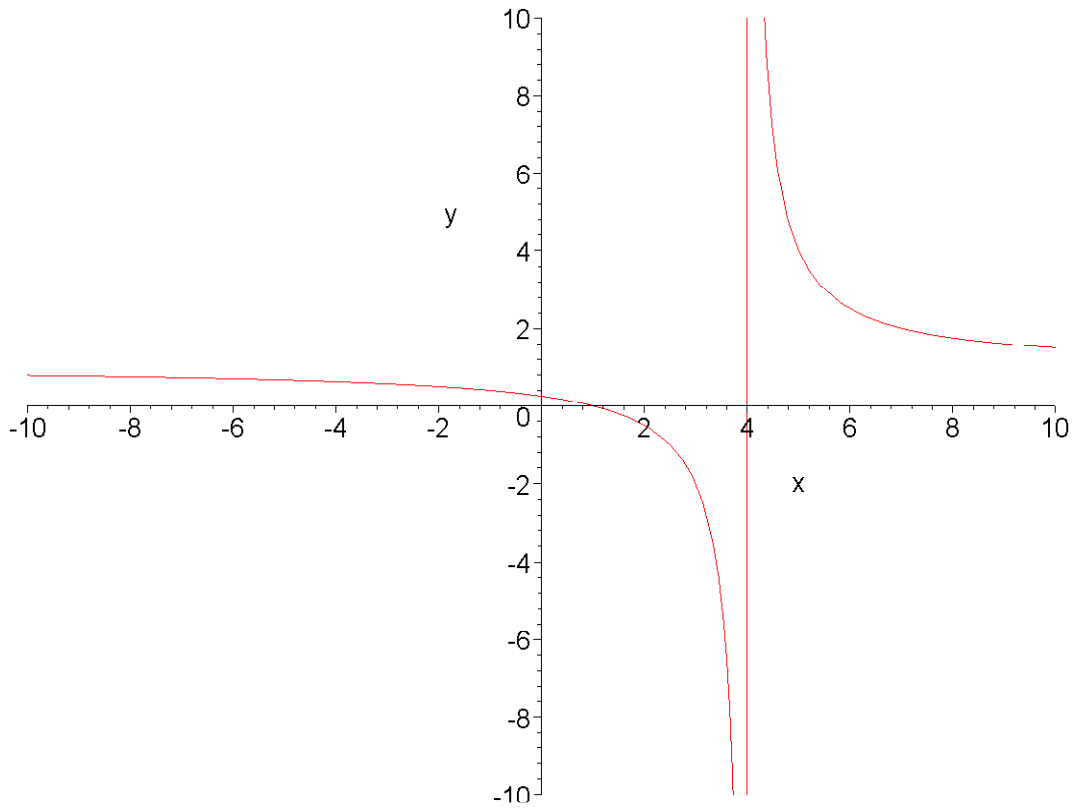
Page 3 ex 11

```
> plot((x^2-9)/(x+1),x= -10 .. 10, y=-10..10);
```



Page 3 ex 12

```
> plot((x-1)/(x-4),x= -10 .. 10, y=-10..10);
```



>

page 3 III ex 1

```
> g(x):= (sin(x))/x; Diff(g(x),x)= diff(g(x),x);
```

$$g(x) := \frac{\sin(x)}{x}$$

$$\frac{\partial}{\partial x}\frac{\sin(x)}{x} = \frac{\cos(x)}{x} - \frac{\sin(x)}{x^2}$$

ex 2

```
> g:= x -> x^(2/3);
  Diff(g(x),x)= diff(g(x),x);
```

$$g := x \to x^{(2/3)}$$

$$\frac{\partial}{\partial x} x^{(2/3)} = \frac{2}{3}\frac{1}{x^{(1/3)}}$$

ex 3

```
> g:= x -> x^2/2-4/x^2;
  Diff(g(x),x)= diff(g(x),x);
```

$$g := x \to \frac{1}{2}x^2 - \frac{4}{x^2}$$

$$\frac{\partial}{\partial x}\left(\frac{1}{2}x^2-\frac{4}{x^2}\right)=x+\frac{8}{x^3}$$

ex4

```
> g(x) := x^4-x^3-3*x^2;
  Diff(g(x),x)= diff(g(x),x);
```

$$g(x) := x^4 - x^3 - 3\,x^2$$

$$\frac{\partial}{\partial x}(x^4 - x^3 - 3\,x^2) = 4\,x^3 - 3\,x^2 - 6\,x$$

page 4 ex5

```
> g(x) := sin(x)*cos(x);Diff(g(x),x)= diff(g(x),x);
```

$$g(x) := \sin(x)\cos(x)$$

$$\frac{\partial}{\partial x}\sin(x)\cos(x) = \cos(x)^2 - \sin(x)^2$$

ex6

```
> g(x) := tan(x)*(4-x^2);Diff(g(x),x)= diff(g(x),x);
```

$$g(x) := \tan(x)\,(4 - x^2)$$

$$\frac{\partial}{\partial x}\tan(x)\,(4 - x^2) = (1 + \tan(x)^2)\,(4 - x^2) - 2\tan(x)\,x$$

Section IV

ex1

```
> s(t):= 6*t-16*t^2;v(t):=Diff(s(t),t);v:=t -> eval(diff(s(t),t));
  eq:= v(t);
```

$$s(t) := 6\,t - 16\,t^2$$

$$v(t) := \frac{\partial}{\partial t}(6\,t - 16\,t^2)$$

$$v := t \to \mathrm{eval}(\mathrm{diff}(s(t), t))$$

$$eq := 6 - 32\,t$$

ex b

```
> solve(eq,t);
```

$$\frac{3}{16}$$

ex c

```
> eq:=s(t);solve(eq,t);
```

$$eq := 6\,t - 16\,t^2$$

$$0, \frac{3}{8}$$

Section V ex1

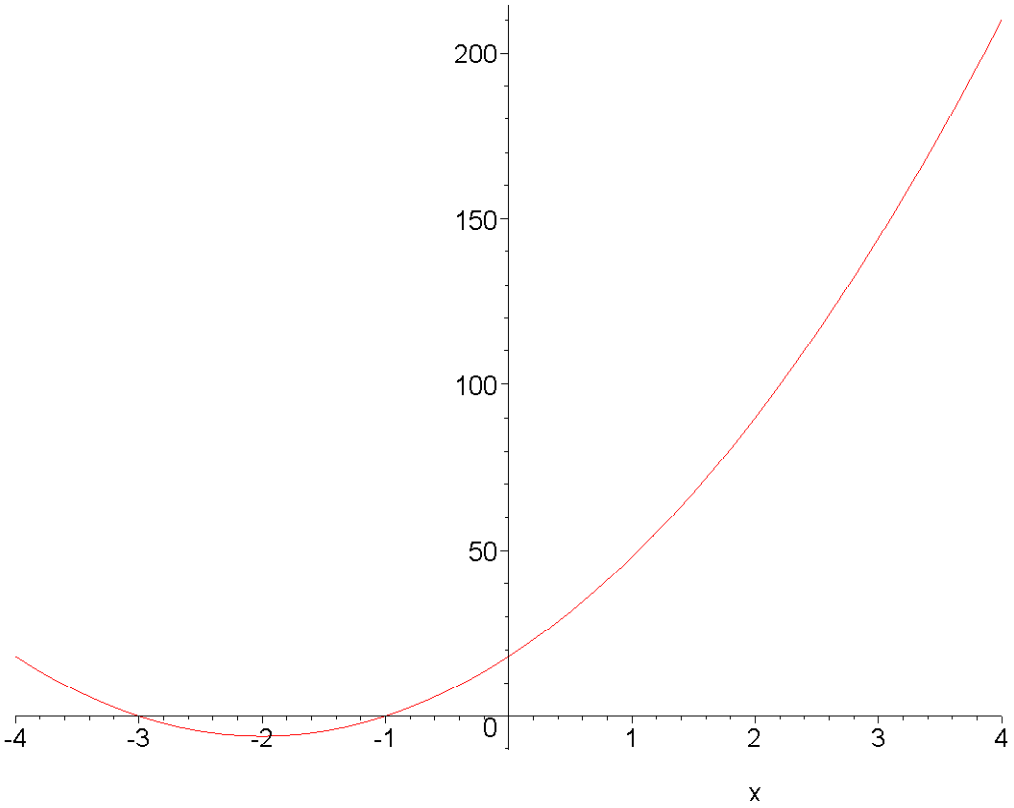
```
> f:=x->2*x^3+12*x^2+18*x+1;v(x):=Diff(f(x),x);v(x):=
  eval(diff(f(x),x));
```

$$f := x \to 2x^3 + 12x^2 + 18x + 1$$

$$v(x) := \frac{\partial}{\partial x}(2x^3 + 12x^2 + 18x + 1)$$

$$v(x) := 6x^2 + 24x + 18$$

```
> plot(v(x),x= -4 .. 4 );
```




```
> eq:=v(x);solve(eq,x);
```

$$eq := 6x^2 + 24x + 18$$

$$-1, -3$$

```
>
```

Section VI ex1

```
> f:=x->x^3-x^2;v(x):=Diff(f(x),x);v(x):= eval(diff(f(x),x));

>
>
```

$$f := x \to x^3 - x^2$$

$$v(x) := \frac{\partial}{\partial x}(x^3 - x^2)$$

$$v(x) := 3x^2 - 2x$$

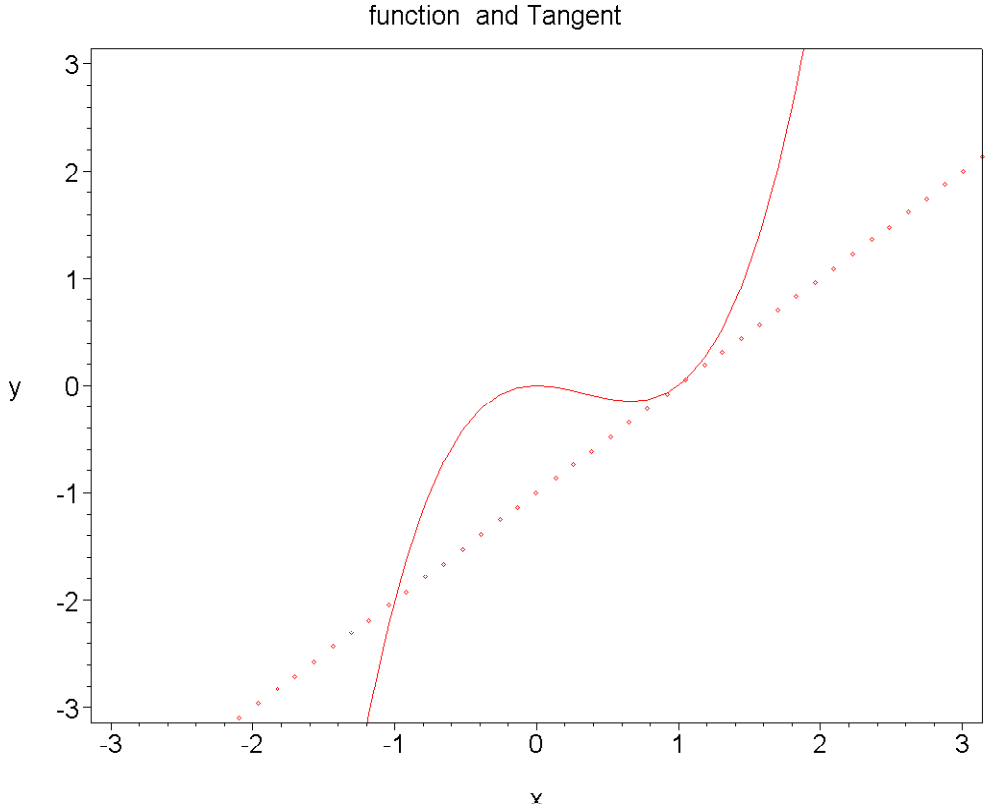
```
> m:= eval(%,x=1);
```

$$m := 1$$

```
> lin(x):=m*(x-1)+f(1);
```

$$\mathrm{lin}(x) := x - 1$$

```
> F:= plot(f(x),x=-Pi..Pi,y=-Pi..Pi,style=line): G:=
  plot(lin(x),x=-Pi..Pi,y=-Pi..Pi,style=point):
  display({F,G},axes=boxed,title=`function  and Tangent`);
```



Section IIII ex1

```
> f:=x-> sqrt(x) +1;v(x):=Diff(f(x),x);v(x):= eval(diff(f(x),x));

>
>
```

$$f := x \to \sqrt{x} + 1$$

$$\mathrm{v}(x) := \frac{\partial}{\partial x}(\sqrt{x} + 1)$$

$$\mathrm{v}(x) := \frac{1}{2}\frac{1}{\sqrt{x}}$$

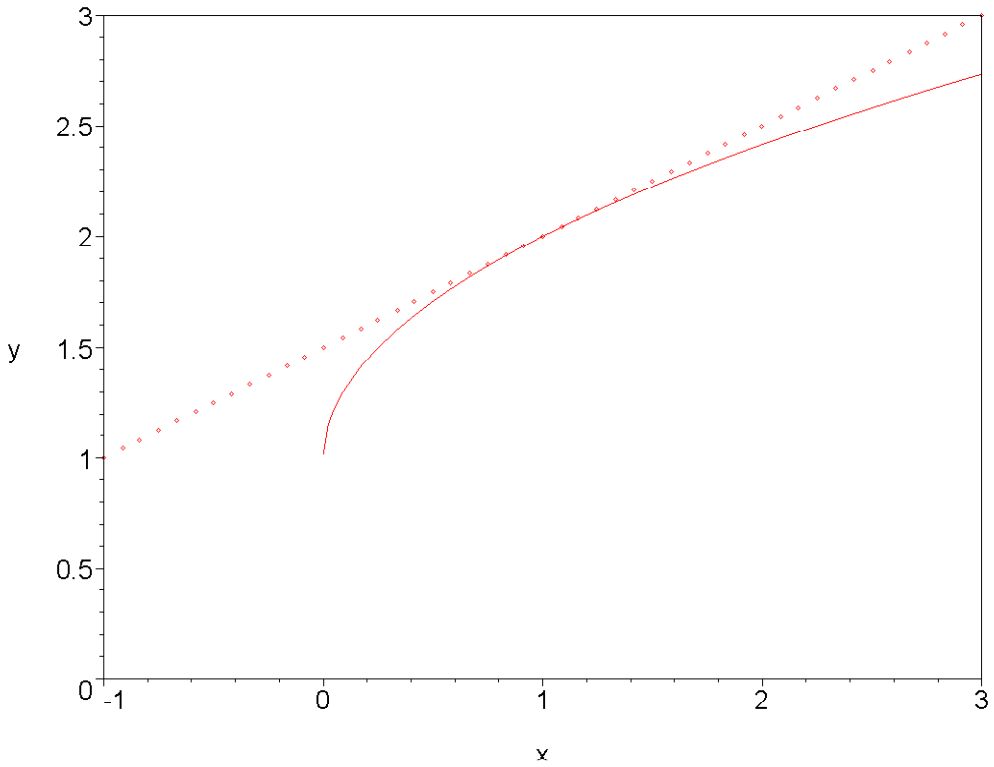
```
> m:= eval(%,x=1);
```

$$m := \frac{1}{2}$$

```
> lin(x):=m*(x-1)+f(1);
```

$$\mathrm{lin}(x) := \frac{1}{2}x + \frac{3}{2}$$

```
> F:= plot(f(x),x=-1..3,y=0..3,style=line): G:=
  plot(lin(x),x=-1..3,y=0..3,style=point):
  display({F,G},axes=boxed,title=`function  and Tangent`);
```



Section III

ex1

```
> s(t):= t^3-3*t^2-24*t;v(t):=Diff(s(t),t);v:=t ->
  eval(diff(s(t),t)): eq:= v(t);
```

$$\mathrm{s}(t) := t^3 - 3\,t^2 - 24\,t$$

$$\mathrm{v}(t) := \frac{\partial}{\partial t}(t^3 - 3\,t^2 - 24\,t)$$

$$eq := 3\,t^2 - 6\,t - 24$$

ex b

```
> solve(eq,t);
```

$$4, -2$$

```
>
```