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Math. 2673 sample Exam II

1. Evaluate as indicated:
   a. Compute the limit along any line and state any conclusion:
   $\lim_{(x,y)->(0,0)} \frac{x^2y^2}{x^2+2y^2}$
   b. Compute the limit along any line and state any conclusion:
   $\lim_{(x,y)->(0,0)} \frac{2xy}{x^2+y^2}$
2. If $u = x^y$, show that
   $\frac{x}{y}\frac{\partial u}{\partial x} + \frac{1}{\ln x}\frac{\partial u}{\partial y} = 2u$
3. Find the maximum rate of change of $f(x,y) = x^2y + \sqrt{(}y)$ at the point $(2,1)$. In which direction does this max occur?
4. If $z = \cos(xy) + y\cos(x)$, where $x = u^2 + v$ and $y = u - v^2$, use the Chain Rule to find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$
5. For $f(x,y) = x\exp(y) + \cos(xy)$ and $P = (2,0)$.
   a.) Find the rate of change in the $(3,-4)$ direction.
   b.) Find the gradient vector at P.
   c.) IN what direction is the maximum rate of change?
   d.) What is the maximum rate of change at P?
   e.) What is the direction of zero change at P, and what is that change at P?
6. For $x^2 + y^2 + z - 9 = 0$ and $P = (1,2,4)$.
   a.) Find the equation of the tangent plane at P.
7. For the function. $f(x,y) = -x^2 + xy - 2y - 2x - y^2 + 4$
   a.) Find all critical points.
8. If $w = xy + z$ and $x = \cos(t), y = \sin(t), z = t$.
   a.) use **(ONLY)** the Chain Rule to find $\frac{dw}{dt}$
9. If $z = x^2 + y^2$, where $x = u + v$ and $y = u - v$,
   a. Compute $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}$.
   b. use the Chain Rule to find $\frac{\partial z}{\partial u}$
10. For the function. $f(x,y) = -x^2 + xy - 2y - 2x - y^2 + 4$
    a.) Find all critical points.
11.) Find the equation of the tangent plane at the point $P = (-3,1,-3)$ to the ellipsoid $x^2/9 + y^2 + z^2/9 = 3$

a.) Second consider $k = f(x, y, z)$ and use the formula $0 = \frac{\partial f}{\partial x}(x - x_o) + \frac{\partial f}{\partial y}(y - y_o) + \frac{\partial f}{\partial z}(z - z_o)$ where all the partials are evaluated at P_o. Hopefully you get the same answer.

12.) For $u = x^4 * y + y^2 * z^3$ and $x = r * s * e^t$, $y = r * s^2 * e^{-t}$ and $z = r * s * \sin(t)$ find $\frac{\partial u}{\partial s}$. find the other 2

13.) Temperature is given by $T(x, y, z) = \frac{80}{1 + x^2 + 2y^2 + 3z^2}$ where $\bigtriangledown T = \frac{\partial T}{\partial x}i + \frac{\partial T}{\partial y}j + \frac{\partial T}{\partial z}k$

a.) In what direction does T increase most rapidly?

b.) What is that increase?

14.) For $z = x^2 + 3xy - y^2$ find the tangent plane at (2,3).

15.) Find the maximum rate of change of $f(x, y) = x^2y + \sqrt{(}y)$ at the point $(2, 1)$. In which direction does this max occur?

16.) If $z = \cos(xy) + y\cos(x)$, where $x = u^2 + v$ and $y = u - v^2$, use the Chain Rule to find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$